COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1938

VOLUME XVIII

№ 2

IMPERIAL MYCOLOGICAL
24 FEB 1938
INSTITUTE

EDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

AVIS AUX AUTEURS

Les «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS» paraissent tous les dix jours, neuf numéros formant un volume d'environ 500 pages pourvu d'une table des matières.

Les «Comptes Rendus de l'Académie des Sciences de l'URSS» publient des communications courtes précisant l'essentiel des travaux plus volumineux en préparation. Ces articles, ne dépassant pas comme règle dix mille signes typographiques, sont donnés en l'une des trois langues: français, anglais, allemand. Ils embrassent les sciences mathématiques, physiques, naturelles et techniques, reproduisant les résultats de recherches scientifiques actuellement en cours ou à peine achevées qui représentent les plus récentes données de l'investigation scientifique.

Les auteurs ont droit chacun à 25 extraits gratuits.

Les manuscrits doivent être adressés à la Rédaction des «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS»—Moscou, oulitza Koujbysheva, 8.

15 Janvier

COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

COMITÉ DE RÉDACTION

A. Borissiak, de l'Académie, **A. Fersman**, de l'Académie, **A. Frumkin**, de l'Académie, **A. Richter**, de l'Académie, **S. Wawilow**, de l'Académie

NOUVELLE SÉRIE

6-me année

Paraissant tous les dix jours

1938

VOLUME XVIII, № 2

TABLE DES MATIÈRES

Pages

MATHÉMATIQUES

S. Sobolev (Soboleff), membre correspondant de l'Académie. Sur une classe d'équations intégrodifférentielles à plusieurs variables indépendantes. II 75

K. Persidskij. Über das Gesetz der grossen Zahlen 81

ASTRONOMIE

K. Brodovitskij. Sur l'analyse statistique du rythme des régularités latentes de fond, dont les manifestations superficielles observables sont altérées par quelques perturbations aléatoires inconnues 85

PHYSIQUE

I. M. Goldman. Breakdown of Compressed Nitrogen in Small Gaps . . . 89

E. Gross. The Splitting-up of the Rayleigh Line and the Acoustic Spectrum of Crystals 93

CHIMIE

P. Petrenko-Kritschenko, korrespondierendes Mitglied der Akademie. Über das Gesetz der Periodizität 95

AGROPHYSIOLOGIE

M. V. Gorlenko. Resistance of Winter Wheats to Black Bacteriosis (*Bacterium translucens* var. *undulosum*, S. J. R.) 97

PHYSIOLOGIE VÉGÉTALE

A. A. Isakova and V. A. Andrejev. On the Excretion of Nitrogenous Compounds by the Roots of Leguminous Plants (Lupine) and the Conversion of These Compounds in the Presence of Bacteria 101

M. Abakumova. On the Problem of Photoperiodical and Temperature After-Effects 105

ANATOMIE DES PLANTES

V. G. Aleksandrov and **O. G. Aleksandrova.** On the Flintiness and Flouriness of Wheat Endosperm 111

MÉCANIQUE DU DÉVELOPPEMENT

N. G. Chlopin. Attempt at an Experimental Analysis of Auxiliary Components of the Nervous System 115

J. A. Vinnikov. Growth and Transformation *in vitro* of Myoneural Elements (Sphincter and Dilatator of the Iris) 119

V. P. Michailov. Growth and Transformation *in vitro* of the Covering Cells of the Vascular Plexuses of the Brain 121

N. N. Kannegisser. Growth and Transformation *in vitro* of the Elements of the Pia Mater 123

G. A. Schmidt. Ektoderm der Anurengastrula unter Induktionswirkung abgetöteter Organisatoren und Organteile von Amphibien und Mammalien 125

Rédacteur en chef A. Fersman. Rédacteur S. Bernstein.

En composition 15/XII 1937. En impression 15/I 1938. Nombre de feuilles d'impression 3 1/2. Nombre de caractères par feuille d'impression 65 000. Pages 73—128. Format du papier 72×110 cm. Tirage 1300. Moscou. Glavlit B 36114. Numéro d'édition 776. Numéro typographique 1716.

Imprimerie № 16. Moscou. Trekhproudnij péréoulok, 9.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

MATHÉMATIQUES

SUR UNE CLASSE D'ÉQUATIONS INTÉGRODIFFÉRENTIELLES À PLUSIEURS VARIABLES INDÉPENDANTES. II

Par S. SOBOLEV (SOBOLEFF), membre correspondant de l'Académie

Dans notre premier article ayant le même titre (C. R. Acad. Sci. URSS, XVII, № 9, 1937) nous avons étudié quelques équations intégro-différentielles d'un type spécial. Nous avons introduit la notion d'indices de régularité ρ_1 et ρ_2 d'une telle équation. Nous avons établi que les équations aux indices positifs, sont résolubles au moyen de la méthode des approximations successives.

Dans la présente note nous allons indiquer un moyen pour résoudre l'équation:

$$\left.\begin{aligned} u(x_1^{(0)}, \dots x_n^{(0)}, t^{(0)}) &= f(x_1^{(0)}, \dots x_n^{(0)}, t^{(0)}) + \\ + \int_{0 \leqslant t^{(1)} \leqslant t^{(0)} - r^{(01)}} \overbrace{\cdots}^{n+1} \int & \sum K_{\alpha_0 \dots \alpha_n} \frac{\partial^l u}{\partial t^{(1)\alpha_0} \partial x_1^{(1)\alpha_1} \dots \partial x_n^{(1)\alpha_n}} dx_1^{(1)} \dots dx_n^{(1)} dt^{(1)} \\ u &= f + Lu \end{aligned}\right\} \quad (1)$$

où

quand

$$\rho_2 = 0. \quad (2)$$

Dans ce cas il faut faire quelques hypothèses supplémentaires sur la nature de l'opération.

D'ailleurs nous donnons des exemples d'après lesquels on voit qu'en général l'équation (1) n'a pas de solutions si $\rho_1 = 0$ ou $\rho_2 < 0$.

Considérons le cas particulier où n est un nombre impaire $n = 2s - 1$.

Nous considérons le cas où l'opération intégrodifférentielle L ne contient que des dérivées dont l'ordre ne surpasse pas $\frac{n+1}{2} = S$ et où par suite le second degré μ pour tous les noyaux qui correspondent aux dérivées d'ordre s est non négatif. Toutes les dérivées d'ordre supérieur peuvent être annulées au moyen de l'intégration par parties, donc, notre hypothèse ne restreint pas la généralité.

Considérons les variables:

$$\xi_i^{(01)}, \eta_i^{(01)}, \eta^{(01)}, \vartheta_1^{(01)}, \dots \vartheta_{n-2}^{(01)}, \varphi^{(01)} \quad (3)$$

introduites dans la note précédente.

Soit $K_{p_0 \dots p_n}(x_1^{(0)}, \dots x_n^{(0)}, t^{(0)}; \xi_0^{(01)}, \eta^{(01)}, \dots, \varphi^{(01)}$ un noyau quelconque pour lequel $p_0 + p_1 \dots + p_n = s$. Considérons les fonctions:

$$\overline{K}_{p_0 \dots p_n}(x_1^{(0)}, \dots x_n^{(0)}, t^{(0)}; \xi_0^{(01)}, \dots \xi_n^{(01)}) =$$

$$= \overline{\overline{K}}_{p_0 \dots p_n}(x_1^{(0)}, \dots x_n^{(0)}, t^{(0)}; \xi_0^{(01)}, \eta^{(01)}, \vartheta_1^{(01)}, \dots \vartheta_{n-2}^{(01)}, \varphi^{(01)}) =$$

$$= K_{p_0 \dots p_n}(x_1^{(0)}, \dots x_n^{(0)}, t^{(0)}; x_1^{(1)}, \dots x_n^{(1)}, t^{(1)}).$$

Nous disons que le noyau $K_{p_0 \dots p_n}$ est un noyau du type invariant si les conditions suivantes sont vérifiées:

a) Pour $\eta^{(01)} \to 1$ le noyau $\overline{\overline{K}}_{p_0 \dots p_n}$ tend vers une limite déterminée:

$$\lim_{\eta^{(01)} \to 1} \overline{\overline{K}}_{p_0 \dots p_n} = K_{(1) p_0 \dots p_n}(x_1^{(0)}, \dots x_n^{(0)}, t^{(0)}; \xi_0^{(01)}, \vartheta_1^{(01)}, \dots \vartheta_{n-2}^{(01)}, \varphi^{(01)}). \quad (4)$$

b) Soit

$$\psi(\eta^{(01)}) = \begin{cases} 1; & \eta \geqslant \frac{2}{3}, \\ \frac{1}{2} + \frac{2}{\pi} \operatorname{arctg} \operatorname{sh} \dfrac{\eta - \frac{1}{2}}{\left(\eta - \frac{1}{3}\right)\left(\frac{2}{3} - \eta\right)}; & \frac{1}{3} \leqslant \eta \leqslant \frac{2}{3}, \\ 0; & \eta \leqslant \frac{1}{3}. \end{cases} \quad (5)$$

Cette fonction est continue ainsi que toutes ses dérivées dans l'intervalle $0 \leqslant \eta \leqslant 1$.

Alors le noyau $K_{p_0 \dots p_n}$ peut être présenté sous la forme:

$$\overline{\overline{K}}_{p_0 \dots p_n} = \overline{\overline{K}}_{(1) p_0 \dots p_n} \psi(\eta^{(01)}) + \overline{\overline{K}}_{(2) p_0 \dots p_n}, \quad (6)$$

où le noyau $\overline{\overline{K}}_{(1) p_0 \dots p_n} \psi(\eta^{(01)})$ possède les degrés $\lambda_1 > -s$ et $\mu_1 = 0$, et le noyau $\overline{\overline{K}}_{(2) p_0 \dots p_n}$ possède les degrés $\lambda_2 > -s$ et $\mu_2 > 0$.

Nous dirons que $\overline{\overline{K}}_{(1) p_0 \dots p_n} \psi(\eta^{(01)})$ est la partie principale du noyau $K_{p_0 \dots p_n}$ et le noyau $\overline{\overline{K}}_{(2) p_0 \dots p_n}$ est le reste de ce noyau.

Si nous omettons dans L toutes les dérivées d'ordre inférieur à s et si nous remplaçons les facteurs des dérivées d'ordre s par leurs parties principales, nous obtenons une autre opération $\overset{*}{L}$. $\overset{*}{L}$ sera nommée la partie principale de l'opération L.

Nous avons le théorème suivant.

Théorème I. *Pour toutes les fonctions f de la classe $\varepsilon_1 > N$ le produit de deux opérations $L_1 L_2 = L_3$ du type invariant est une opération du type invariant. La partie principale $\overset{*}{L}_3$ de l'opération L_3 dépend seulement des opérations $\overset{*}{L}_1$ et $\overset{*}{L}_2$.*

Si nous désignons par

$$K^{(02)}_{p_0 \dots p_n}(x_1^{(0)} \dots x_n^{(0)}, t^{(0)}; x_1^{(2)}, \dots x_n^{(2)}, t^{(2)})$$

les noyaux de l'opération L_1 pour lesquels $p_0 + \dots + p_n = s$, par

$$K^{(21)}_{q_0 \dots q_n}(x_1^{(2)}, \dots x_n^{(2)}, t^{(2)}; x_1^{(1)}, \dots x_n^{(1)}, t^{(1)})$$

les noyaux de l'opération L_2 et par

$$K^{(01)}_{(1) r_0 \dots r_n}(x_1^{(0)} \dots x_n^{(0)}, t^{(0)}; \xi_0^{(01)}, \vartheta_1^{(01)}, \dots \vartheta_{n-2}^{(01)}, \varphi^{(01)}) \cdot \psi(\eta^{(01)})$$

ceux de l'opération $\overset{*}{L}_3$, nous obtiendrons la formule fondamentale suivante:

$$\overline{\overline{K}}^{(01)}_{(1)\,q_0 \ldots q_n} = \sum_{p_0 + \ldots + p_n = s} \sin \vartheta_1^{(01)p_2 + \ldots + p_n} \sin \vartheta_2^{(01)p_3 + \ldots + p_n} \ldots$$

$$\ldots \sin \varphi^{(01)p_n} \cos \vartheta_1^{(01)p_1} \ldots \cos \varphi^{(01)p_{n-1}} \times$$

$$\times \frac{\xi_0^{(01)s} \pi^{s-1} s}{2^s} \int_{-1}^{+1} (1 - \rho_2^2)^{s-1} \overline{\overline{K}}^{(02)}_{(1)p_0 \ldots p_n} \times$$

$$\times \Big(x_1^{(0)}, \ldots x_n^{(0)}, t^{(0)}; \xi_0^{(01)} \frac{(1-\rho_2)}{2}, \vartheta_1^{(01)}, \ldots \varphi^{(01)} \Big) \times$$

$$\times \overline{\overline{K}}^{(21)}_{(1)\,q_0 \ldots q_n} \Big(x_1^{(0)} - \frac{\xi_0^{(01)}(1-\rho_2)}{2} \cos \vartheta_1^{(01)}, \ldots x_2^{(0)} -$$

$$- \frac{\xi_0^{(01)}(1-\rho_2)}{2} \sin \vartheta_1^{(01)} \cos \vartheta_2^{(01)}, \ldots x_n^{(0)} - \frac{\xi_0^{(01)}(1-\rho_2)}{2} \sin \vartheta_1^{(01)} \ldots$$

$$\ldots \sin \varphi^{(01)}, t^{(0)} - \frac{\xi_0^{(01)}(1-\rho_2)}{2}; \xi_0^{(01)} \frac{(1+\rho_2)}{2}, \vartheta_1^{(01)}, \ldots \varphi^{(01)} \Big) d\rho_2. \qquad (7)$$

Cette formule permet de passer à la résolution de nos équations intégrodifférentielles.

Indiquons une proposition évidente:

Si dans l'équation (1) l'opération L est du type invariant à partie principale nulle, cette équation peut être résolue par la méthode des approximations successives.

Revenons à l'équation (1). Cherchons la fonction inconnue u sous la forme:

$$u = \varphi + G\varphi, \qquad (8)$$

où $G\varphi$ est une opération intégrodifférentielle du type invariant.

Si nous remplaçons dans (1) la fonction u par son expression (8) nous aurons:

$$\varphi + G\varphi = f + L\varphi + LG\varphi$$

ou

$$\varphi = f + (L - G + LG)\,\varphi. \qquad (9)$$

Si nous choisissons l'opération G de telle manière que

$$\overset{*}{L} - \overset{*}{G} + \overset{*}{L}\overset{*}{G} = 0 \qquad (10)$$

l'équation (9) peut être résolue au moyen des approximations successives et par suite nous obtenons la solution de (1).

Grâce à la formule (7) l'équation (10) n'est qu'un système d'équations intégrales (et non pas intégrodifférentielles) pour les parties principales des noyaux de G. Le lemme suivant aura lieu:

Lemme 1. *Le système d'équations qu'on obtient de* (10) *peut être résolu par la méthode des approximations successives. Le produit de chaque fonction qui entre dans la solution de ce système par* $\psi(\eta^{(01)})$ *nous donne un noyau régulier de degrés* $\lambda > -s$ *et* $\mu = 0$.

Nous allons établir le théorème suivant.

Théorème II. *La solution de l'équation* (1) *est unique si l'on se borne aux fonctions de la classe* $\varepsilon_1 > -1$.

Soit u une solution quelconque. Substituons cette solution dans (8) et cherchons la fonction φ de cette équation. Comme nous venons de démontrer cette équation admet toujours une solution. Donc, chaque solution u de (1) est représentable sous la forme (8). Des équations (1) et (8) résulte (9). En tenant compte de ce que la solution de l'équation (9) est unique, nous voyons que la solution de (1) est aussi unique.

Passons à la solution du problème pour n paire. Dans ce but nous profiterons de la «méthode de descente» de M. Hadamard. Soit

$$u = f + Lu \tag{1}$$

l'équation en question.

Étudions le cas, où l'opération L ne contient pas de dérivées d'ordre supérieur à $\frac{n}{2}+1$, et où les degrés de ses noyaux μ satisfont à l'inégalité:

$$\mu > -\frac{1}{2}.$$

L'indice ρ_1 est supposé positif.

Alors les degrés des noyaux qui sont facteurs des dérivées d'ordre $\frac{n}{2}+1$ seront

$$\lambda > -\frac{n}{2} \text{ et } \mu \geqslant \frac{1}{2}.$$

Au lieu de l'équation (1) nous considérons une autre équation qui contient une nouvelle variable indépendante $y = x_{n+1}$ et dans laquelle chaque noyau $K_{p_0 \dots p_n}$ est remplacé par le noyau:

$$\frac{K_{p_0 \dots p_n}}{\sqrt{\xi_0^{(01)2} - (y^{(0)} - y^{(1)})^2}}.$$

Nous avons déduit le problème au cas déjà examiné.

Donnons deux exemples qui montrent qu'en général l'équation intégrodifférentielle du type considéré n'a pas de solution pour $\rho_1 = 0$ et $\rho_2 < 0$.

Exemple I.

Soit $u(x_1^{(0)}, \dots x_n^{(0)}, t^{(0)})$ une fonction de la classe $\varepsilon_1 > N$. Introduisons le système de coordonnées polaires dans l'espace à $(n+1)$ dimensions:

$$\left.\begin{aligned}
\xi_0^{(01)} &= \Xi \cos\theta, \\
\xi_i^{(01)} &= \Xi \sin\theta \sin\vartheta_1^{(01)} \dots \sin\vartheta_{i-1}^{(01)} \cos\vartheta_i^{(01)}, \\
\xi_{n-1}^{(01)} &= \Xi \sin\theta \sin\vartheta_1^{(01)} \dots \sin\vartheta_{n-2}^{(01)} \cos\varphi^{(01)}, \quad (i = 1, 2 \dots n-2), \\
\xi_n^{(01)} &= \Xi \sin\theta \sin\vartheta_1^{(01)} \dots \sin\vartheta_{n-2}^{(01)} \sin\varphi^{(01)}.
\end{aligned}\right\} \tag{11}$$

Nous aurons

$$u(x_1^{(0)}, \dots x_n^{(0)}, t^{(01)})\, u|_{\Xi=0} = \int_0^{\frac{t^{(0)}}{\cos\theta}} \frac{\partial^k u}{\partial \Xi^k} \frac{\Xi^{k-1}}{(k-1)!}\, d\Xi. \tag{12}$$

En multipliant cette identité par $e^{\frac{1}{\cos\theta-\cos\frac{\pi}{4}}}$ et en intégrant par rapport à θ, $\vartheta_i^{(01)}$, $\varphi^{(01)}$, dans le domaine

$$0\leqslant\theta\leqslant\frac{\pi}{4};\quad 0\leqslant\vartheta_i^{(01)}\leqslant\pi\ (i=1,2\ldots n-2);\quad 0\leqslant\varphi^{(01)}\leqslant 2\pi \tag{13}$$

nous obtenons, après avoir fait quelques calculs, une nouvelle identité:

$$u\ (x_1^{(0)},\ldots x_n^{(0)},\ t^{(0)}) =$$

$$=\overbrace{\int\ldots\int}^{n+1}_{(\leqslant t^{(1)}\leqslant t^{(0)}-r^{(01)})}\ \sum_{p_0+\ldots+p_n=k} K_{p_0\ldots p_n}\frac{\partial^k u\ (x_1,\ldots x_n)}{\partial t^{(1)p_0}\ldots\partial x_n^{(1)p_n}}\ dx_1^{(1)}\ldots dx_n^{(1)}dt^{(1)}=\mathfrak{M}u, \tag{14}$$

où les noyaux $K_{p_0\ldots p_n}$ ont les degrés $\lambda=k-n-1$ et $\mu=+\infty$.

L'équation (14) a une infinité de solutions.

Au contraire, l'équation:

$$u=f+\mathfrak{M}u, \tag{15}$$

où f est de la classe $\varepsilon_1>0$, n'aura aucune solution, car $\mathfrak{M}u$ est identique à u.

Exemple II.

Comme second exemple nous considérons une équation pour $n=1$. Mais la méthode de descente permet d'en déduire une quantité d'autres exemples.

Considérons l'équation

$$u\ (x^{(0)},t^{(0)})=f(x^{(0)}-t^{(0)})+\int\limits_{0\leqslant t^{(1)}\leqslant t^{(0)}-|x^{(0)}-x^{(1)}|}\int K\ (x^{(0)},t^{(0)};x^{(1)},t^{(1)})\frac{\partial^l u\ (x^{(1)},\ t^{(1)})}{\partial x^{(1)^l}}dx^{(1)}dt^{(1)}, \tag{16}$$

où le noyau K est déterminé par la formule:

$$K\ (x^{(0)},t^{(0)};x^{(1)},t^{(1)})=$$

$$=\begin{cases}\dfrac{(\xi_0^{(01)}-A)^2}{(\xi_0^{(01)}-A)^2-\delta^2}\left(\xi_0^{(01)}-\xi_1^{(01)}\right)^{l-\frac{1}{m}-1}\psi\left(\dfrac{1}{2}+\dfrac{1}{2}\dfrac{\xi_1^{(01)}}{\xi_0^{(01)}}\right); & |\xi_0^{(01)}-A|<\delta,\\ 0, & |\xi_0^{(01)}-A|>\delta.\end{cases} \tag{17}$$

m est un nombre entier quelconque.

Considérons le système d'intervalles α_i:

$$(i-1)\ (A+\delta)\leqslant t^{(0)}\leqslant i\,(A-\delta).$$

On peut démontrer que dans chaque intervalle α_i la solution de (16) dépend seulement de la différence $x^{(0)}-t^{(0)}$.

Si nous posons dans α_i:

$$u=u_i\ (x^{(0)}-t^{(0)}),$$

nous pouvons obtenir la formule récurrente suivante:

$$u_{i+m}\ (\xi)=f\ (\xi)+\frac{\partial u_i(\xi)}{\partial\xi}+\int_a^b\Omega\ (\eta)\ u_i\ (\xi+\eta)\ d\eta, \tag{18}$$

où $a>0$, $b>0$ et le noyau $\Omega\ (\eta)$ a une infinité de dérivées continues.

Il s'en suit que si la fonction f admet seulement un nombre fini de dérivées il existe un intervalle α_i dans lequel la solution n'existe pas.

On peut choisir A et δ de telle façon que l'équation (16) n'ait pas de solutions dans un intervalle des valeurs de t aussi petit que l'on veut.

Les indices de régularité de (16) seront $\rho_1 = \infty$ et $\rho_2 = -\frac{1}{m}$.

Notre théorème est démontré.

Institut mathématique V. A. Stekloff.
Académie des Sciences de l'URSS.
Moscou.

Manuscrit reçu
le 26. XI. 1937.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

MATHEMATIK

ÜBER DAS GESETZ DER GROSSEN ZAHLEN

Von K. PERSIDSKIJ

(Vorgelegt von S. N. Bernstein, Mitglied der Akademie, d. 3. XII. 1937)

Es seien $x_1, x_2, \ldots, x_n, \ldots$ irgendwelche voneinander unabhängige oder abhängige Grössen. Wir nehmen an, dass die mathematischen Erwartungen dieser Grössen existieren:

$$\mathfrak{M}\,|\,x_s\,| = c_s \quad (s = 1, 2, \ldots).$$

Wir wollen sagen, die $\mathfrak{M}\,|\,x_s\,|$ konvergieren gleichmässig (in bezug auf s), falls fur jedes vorgegebene $\alpha > 0$ eine endliche Zahl $L = L(\alpha)$ gibt, derart dass

$$\mathfrak{M}'\,|\,x_s\,| < \alpha \quad (s = 1, 2, \ldots, n, \ldots)$$

gilt, wobei das Symbol $\mathfrak{M}'\,|\,x_s$ den Teil von $\mathfrak{M}\,|\,x_s\,|$ bezeichnet, welchem diejenigen Werte der Grösse x_s entsprechen, die absolut grösser als L sind.

Wir können der Einfachheit halber $\mathfrak{M}x_s = 0$ annehmen. Es sei $1-\eta$ die Wahrscheinlichkeit dafür, dass die Ungleichung

$$|\,x_1 + x_2 + \ldots + x_n\,| < \varepsilon n \tag{1}$$

erfüllt wird. Ferner sei

$$I = \frac{\mathfrak{M}\,|\,x_1 + x_2 + \ldots + x_n\,|}{n} = I_1 + I_2,$$

wobei I_1 den Teil von I bezeichnet, welchem die der Ungleichung (1) genügenden Werte von $(x_1 + x_2 + \ldots + x_n)$ entsprechen, während I_2 den restlichen Teil von I bezeichnet.

Es gilt $I_1 < \varepsilon$. Ohne die Allgemeinheit zu beeinträchtigen, kann man annehmen, dass die Werte von $x_s\,(s = 1, 2, \ldots)$ eine abzählbare Menge $x_s^{(1)}, x_s^{(2)}, \ldots, x_s^{(m)} \ldots$ bilden. Dann kann man die Grösse nI_2 als Summe von Grössen der Gestalt:

$$A_{\alpha_1 \ldots \alpha_s \ldots \alpha_n} = |\,x_1^{(\alpha_1)} + \ldots + x_s^{(\alpha_s)} + \ldots + x_n^{(\alpha_n)}\,| \cdot p_{\alpha_1 \ldots \alpha_s \ldots \alpha_n}$$

darstellen, wobei die Summe sämtlicher $p_{\alpha_1 \ldots \alpha_s \ldots \alpha_n}$ gleich η wegen der Ungleichungen $|\,x_1^{(\alpha_1)} + \ldots + x_s^{(\alpha_s)} + \ldots + x_n^{(\alpha_n)}\,| \geqslant n\varepsilon$ ist. Andererseits ist

$$A_{\alpha_1 \ldots \alpha_s \ldots \alpha_n} \leqslant (\,|\,x_1^{(\alpha_1)}\,| + \ldots + |\,x_s^{(\alpha_s)}\,| + \ldots + |\,x_n^{(\alpha_n)}\,|\,)\; p_{\alpha_1 \ldots \alpha_s \ldots \alpha_n}.$$

Die Summe der Produkte $|x_s^{(\alpha_s)}| p_{\alpha_1 \ldots \alpha_s \ldots \alpha_n}$ ist nicht grösser als $L\eta + \alpha$, da der Teil dieser Summe, welcher den Werten $x_s^{(\alpha_s)}$ entspricht, die dem Modulo nach nicht grösser als L sind, nicht grösser als $L\eta$ ist, während ihr Restteil nicht grösser als $\mathfrak{M}'|x_s|$ und demnach nicht grösser als α ist. Daraus folgt:

$$nI_2 < nL\eta + n\alpha,$$

oder

$$I_2 < L\eta + \alpha.$$

Wir erhalten also den

Hilfssatz 1. *Es gilt*

$$I = \frac{\mathfrak{M}|x_1 + x_2 + \ldots + x_n|}{n} < \varepsilon + L\eta + \alpha.$$

Wir haben den

Satz 1. *Es seien*

$$x_1, x_2, \ldots, x_n, \ldots$$

Grössen, deren mathematische Erwartungen $\mathfrak{M}|x_s|$ *gleichmässig konvergieren. Damit diese Grössen dem Gesetz der grossen Zahlen genügen, ist notwendig und hinreichend die Bedingung*

$$\lim_{n \to \infty} I = 0.$$

Diese Bedingung ist offenbar hinreichend, während ihre Notwendigkeit aus dem Hilfssatz 1 folgt.

Man setze

$$y_s = x_s - z_s,$$

wobei $z_s = 0$ für $|x_s| \leqslant L$, und $z_s = x_s$ für $|x_s| > L$ ist.
Ist

$$\Delta = \frac{\mathfrak{M}'|x_1| + \ldots + \mathfrak{M}'|x_n|}{n},$$

so ist offenbar auch

$$\frac{\mathfrak{M}|z_1| + \ldots + \mathfrak{M}|z_n|}{n} = \Delta \leqslant \alpha.$$

Nun haben wir:

$$\mathfrak{M}|x_1 + \ldots + x_n| - \mathfrak{M}|z_1 + \ldots + z_n| \leqslant \mathfrak{M}|y_1 + \ldots + y_n| \leqslant$$
$$\leqslant \mathfrak{M}|x_1 + \ldots + x_n| + \mathfrak{M}|z_1 + \ldots + z_n|.$$

Daraus folgt:

$$I - \Delta \leqslant \frac{\mathfrak{M}|y_1 + y_2 + \ldots + y_n|}{n} \leqslant I + \Delta.$$

Somit erhalten wir folgenden

Hilfssatz 2. *Sind die* $\mathfrak{M}|x_s|$ *gleichmässig konvergent, so besteht die notwendige und hinreichende Bedingung dafür, dass die Grössen* $x_1, x_2, \ldots, x_n, \ldots$ *dem Gesetze der grossen Zahlen genügen, darin, dass die Wahrscheinlichkeit für das Erfüllen der Ungleichung*

$$|y_1 + y_2 + \ldots + y_n| < n\varepsilon$$

bei genügend grossem L beliebig nahe zu Eins liegt, sobald n genügend gross ist.

Da die Grössen y_s gleichmässig (in bezug auf n) beschränkt sind, so folgt aus dem Hilfssatz 2 der

Satz 2. *Sind die $\mathfrak{M}|x_s|$ gleichmässig konvergent, so besteht die notwendige und hinreichende Bedingung dafür, dass die Grössen $x_1, x_2, \ldots, x_n, \ldots$ dem Gesetz der grossen Zahlen genügen, darin, dass bei jedem vorgegebenem $\beta > 0$, genügend grossem $L \geqslant L(\beta)$ und genügend grossem $n \geqslant n(L, \beta)$ die Ungleichung*

$$\frac{\mathfrak{M}(y_1 + y_2 + \ldots + y_n)^2}{n^2} < \beta \qquad (2)$$

erfüllt ist.

Nun folgt aus $\mathfrak{M} y_s = \mathfrak{M} x_s - \mathfrak{M} z_s = -\mathfrak{M} z_s$ und $|\mathfrak{M} z_s| \leqslant \mathfrak{M}|z_s| < \alpha$ die Ungleichung

$$|\mathfrak{M} y_s| < \alpha.$$

Setzen wir also $u_s = y_s - \mathfrak{M} y_s$, so folgt aus dem Hilfssatz 2, dass dieser Hilfssatz bestehen bleibt, wenn man die Grössen y_s durch die Grössen u_s ersetzt. Daraus folgt, dass bei dieser Ersetzung auch der Satz 2 weiter gilt.

Nun nehmen wir an, die Grössen x_s seien paarweise unabhängig. Dann gilt:

$$\frac{\mathfrak{M}(u_1 + u_2 + \ldots + u_n)^2}{n^2} = \frac{\mathfrak{M} u_1^2 + \mathfrak{M} u_2^2 + \ldots + \mathfrak{M} u_n^2}{n^2} < \frac{(L+\alpha)^2}{n}. \qquad (2')$$

Daraus folgt (1) der

Satz 3. *Sind die Grössen $x_1, x_2, \ldots, x_n, \ldots$ paarweise unabhängig, so ist die Bedingung des gleichmässigen Konvergenz hinreichend dafür, dass die Grössen $x_1, x_2, \ldots, x_n, \ldots$ dem Gesetz der grossen Zahlen genügen.*

Aus diesem Satz folgt der bekannte Satz von Khintchine (2) als Korollar. Denn ist $x_1 \equiv x_2 \equiv \ldots \equiv x_n \equiv \ldots$, so folgt aus der Existenz der Grössen $\mathfrak{M}|x_s|$ auch ihre gleichmässige Konvergenz.

A. Markoff (3) (senior) gab folgendes Theorem:

Gilt

$$\mathfrak{M}|x_s|^{1+\delta} \leqslant c \quad (s = 1, 2, \ldots),$$

wobei $\delta > 0$ und c gewisse konstante Grössen sind, so genügen die paarweise unabhängigen Grössen $x_1, x_2, \ldots, x_n, \ldots$ dem Gesetz der grossen Zahlen.

Das Markoffsche Theorem ist ebenfalls ein Spezialfall des Satzes 3. Denn aus den Ungleichungen

$$\mathfrak{M}'|x_s|^{1+\delta} \geqslant L^{\delta}\mathfrak{M}'|x_s| \quad (s = 1, 2, \ldots, n, \ldots)$$

folgt:

$$\mathfrak{M}'|x_s| \leqslant \frac{c}{L^{\delta}} \quad (s = 1, 2, \ldots, n, \ldots),$$

woraus folgt, dass die $\mathfrak{M}|x_s|$ gleichmässig konvergent sind.

Institut für Mathematik und Mechanik.
Staatsuniversität. Kasan.

Eingegangen
d. 4.XII.1937.

ZITIERTE LITERATUR

(1) Персидский, Ученые записки Каз. университета, **94** (1930). (2) Khintchine, C. R., **188**, 477. (3) А. Марков, Исчисление вероятностей, стр. 129 (1924).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

ASTRONOMIE

SUR L'ANALYSE STATISTIQUE DU RYTHME DES RÉGULARITÉS LATENTES DE FOND, DONT LES MANIFESTATIONS SUPERFICIELLES OBSERVABLES SONT ALTÉRÉES PAR QUELQUES PERTURBATIONS ALÉATOIRES INCONNUES

Par K. BRODOVITSKIJ

(Présenté par V. G. Fessenkov, de l'Académie, le 14. X. 1937)

La résolution de ce problème pourrait avoir des applications dans de différents domaines de la science. Cependant c'est pour l'astrophysique que ce problème présente un intérêt particulier, vu que les processus rythmiques puissants ayant lieu dans les seins profonds des étoiles ou du Soleil sont impénétrables pour des observations directes et le rythme de leurs manifestations superficielles observables (variations de clarté, taches solaires, etc.) est altéré, sans doute, par des perturbations intermédiaires.

Pour expliquer mieux quelques idées, considérons le schème qui suit. Supposons qu'un courrier aérien A suive toujours le même itinéraire fermé passant par notre ville V. Soit A exposé aux éventualités en route qui entraînent des accélérations ou des ralentissements de sa circulation sans amener une avarie. L'attérissage de A dans V se répétera donc d'une façon rythmique. Soit $\sum_y$ l'écart moyen de la durée d'un cycle.

Supposons ensuite que le pilote n'ait pas de montre et conduise son avion sans horaire avec toute vitesse possible et des haltes minima. Les perturbations en route de même que les durées des cycles soient indépendantes. L'attérissage de A dans V soit hors de notre observation. Appelons ce processus rythmique régularité latente de fond (inobservable).

Supposons encore que A apporte chaque fois une lettre adressée à nous. Nous recevons ces lettres successivement. Le parcours des lettres depuis l'aérodrome jusqu'à nous soit aussi exposé aux éventualités en route. Supposons que les durées de ce stade intermédiaire soient indépendantes et que $\sum_z$ soit leur écart moyen.

Désignons par T_i $(i = 1, 2, \ldots, n)$ les époques quand des lettres sont rendues à notre domicile. Appelons ce processus rythmique régularité superficielle observée. Les époques T_i définissent les durées des cycles observés $t_i = T_i - T_{i-1}$ $(i = 2, 3, \ldots, n)$. Désignons encore par σ_y l'écart moyen quadratique des t_i observés, et par $\sum'_y$ la valeur limite de σ_y, si $n \to \infty$. Calculons la régression de l'époque T_i d'un numéro i $(T_i = t' + iT')$. Soit σ_x l'écart moyen quadratique de cette régression et soit $\sum'_x$ défini comme il suit: son carré $\sum'^2_x$ est l'espérance mathémati-

que de la dispersion σ_x^2, le nombre n étant fixe. Par analogie avec $\sum'_x$ nous aurons pour la régularité de fond une estimation $\sum_x$.

Il est avantageux de distinguer deux cas idéaux. Si $\sum_z = 0$ le stade dit intermédiaire n'entraîne aucune perturbation. C'est le cas d'une pulsation pure avec l'écart moyen $\sum_y$. En même temps, le rythme du processus de fond n'y est aucunement dissimulé. Or, si $\sum_y = 0$ ce sera le cas d'une périodicité latente rigoureusement exacte (comme si un «horaire» strict déterminait le processus de fond).

La dispersion observée σ_x^2 est déterminée alors toute entière par les perturbations du stade intermédiaire.

En réalité nous avons des cas mixtes, et il importe de savoir délimiter, combien la dispersion observée est déterminée par les perturbations du stade intérmédiaire, et à quelle mesure elle caractérise la pulsation de fond. Si nous aurons trouvé la valeur $\sum_z$, il nous sera possible d'estimer la limite inférieure de la durée moyenne des stades intermédiaires, qui parfois ne peut être évaluée par aucun des procédés connus.

J'affirme qu'il est possible d'estimer $\sum_y$ et $\sum_z$ quoique ils soient inobservables directement.

En effet, σ_y avec σ_x sont connus. Donc, $\sum'_y \sim \sigma_y$ et $\sum'_x \sim \sigma_x$ sont aussi à peu près connus, les erreurs peuvent être estimées par les méthodes statistiques. Selon les règles bien connues nous avons:

$$\sum\nolimits_x'^2 = \sum\nolimits_x^2 + \sum\nolimits_z^2; \quad \sum\nolimits_y'^2 = \sum\nolimits_y^2 + 2\sum\nolimits_z^2.$$

Dans la première équation nous admettons une inexactitude, ayant, pour simplifier l'expression, supposé que les équations de régression T_i d'un numéro i, obtenues pour la régularité de fond et la régularité superficielle, ne diffèrent l'une de l'autre que par un terme constant qui est la durée moyenne des stades intermédiaires. Au cas des n grands on peut négliger cette inexactitude. L'équation exacte aurait la forme:

$$\sum\nolimits_x'^2 = \sum\nolimits_x^2 + \left(1 - \frac{2}{n}\right)\sum\nolimits_z^2.$$

D'autre part on peut montrer que

$$\sum\nolimits_x^2 = \frac{n^2 - 4}{15\,n}\sum\nolimits_y^2,$$

d'où la recherche des inconnues est élémentaire.

Remarquons que ces équations subsistent, quelles que soient les lois de probabilité de nos variables.

La valeur $R = \frac{\Sigma_z}{\sqrt{\Sigma_z^2 + \Sigma_y^2}}$ nous appelons indicateur de périodicité latente $(0 \leqslant R \leqslant 1)$.

Notons encore que pour $\sum_z = 0$ la succession des cycles observés ne contredit pas la supposition faite de leur indépendance. Donc, on peut considérer les valeurs des coefficients $\frac{\Sigma_z}{\Sigma'_y} = 0$ ou $\frac{\Sigma_y}{\Sigma'_y} = 1$ comme nouveaux critériums de l'absence de liaison, indiquant que la suite donnée de valeurs est une suite indépendante aléatoire. Or la valeur réelle et positive obtenue pour le premier coefficient indiquera la présence de liaison qui sera d'ailleurs caractérisée par ce que l'écart entre la variable et sa moyenne dans un sens sera partiellement équilibré par une plus grande pro-

babilité pour le sens inverse des écarts subséquents (effet de la compensation subséquente partielle).

Je fus chargé par la Direction Générale du Service Hydrométéorologique d'Uzbekistan d'analyser sur la base de ces raisonnements la suite des époques extrêmes des taches solaires, ce que j'ai accompli en 1935.

J'ai obtenu les résultats suivants:

Pour la phase du maximum des taches (1615—1928), $n = 29$; $\sum_y = 1.02 \pm 0.34$; $\sum_z = 1.11 \pm 0.32$ (en années).

Pour la phase susdite (188—1928), consultant les annales dont les témoignages concernant les taches solaires observées à l'œil nu ont été soumis à un choix critique et employés dans notre analyse comme indicateurs approximatifs pour les époques maxima des siècles passés: $n = 103$ d'entre 158; n admis est égal à 126; $\sum_y = 0.60 \pm 0.10$; $\sum_z = 1.25 \pm 0.20$.

Pour la phase du minimum des taches (1610—1933): $n = 30$; $\sum_y = 0.67 \pm 0.30$; $\sum_z = 1.02 \pm 0.23$.

La phase d'extrême a été déterminé d'après la courbe ajustée (pour les valeurs annuelles) de nombres de Wolf. La valeur obtenue immédiatement est $\sum'_y = 1.87 \pm 0.25$. Donc la régularité de fond est plus exacte (quant à la périodicité) que la régularité superficielle observée. La valeur de $\sum_z$, ayant l'ordre d'une année, indique que la durée moyenne du stade intermédiaire des taches compte au moins plusieurs années, ce qui nous permet de conclure que les taches solaires apparaissent des seins profonds.

Évidemment, l'analyse statistique, qui montre que la suite donnée est une suite liée est en même temps incapable de résoudre la question: où le mécanisme régulateur de la liaison se trouve-t-il? Mais tandis que l'hypothèse de l'origine superficielle des taches demande une explication de la nature physique du mécanisme de liaison, l'hypothèse de leur origine dans les seins profonds explique la cohérence de la série d'une manière naturelle par des perturbations aléatoires du stade intermédiaire qui sont superposées sur une pulsation de fond pure.

Au cas où l'on fait déterminer la phase maximum des taches non pas d'après la valeur maximum annuelle des nombres de Wolf, mais d'après la valeur maximum mensuelle de ces nombres, la dispersion $\sum_z^2$ augmente d'une valeur complémentaire remarquable, égale à [0.9 de l'année]^2, ce qui constitue aussi une preuve en faveur de l'origine des taches dans les seins profonds. Le fait d'une régularité de fond plus exacte explique aussi la possibilité d'une extrapolation, par rapport au temps, de l'équation de régression des époques extrêmes des taches dans un intervalle comptant plusieurs siècles [même au delà de deux mille années, d'après E. Sluckij (1)].

Il me paraît raisonnable d'admettre l'ordre de 50 ans pour la durée du stade intermédiaire des taches (nous éviterons ainsi la supposition de grande variations dans la vitesse moyenne du mouvement qui fait transporter le processus formant les taches depuis les régions centrales jusqu'à la surface du Soleil); ce serait important pour développer la théorie de l'origine des taches.

La Direction Générale
du Service Hydrométéorologique
d'Uzbekistan.

Manuscrit reçu
le 26. XI. 1937.

LITTÉRATURE CITÉE

1 E. E. Slutskij, C. R. Acad. Sci. URSS, IV, 1—2 (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

PHYSICS

BREAKDOWN OF COMPRESSED NITROGEN IN SMALL GAPS

By I. M. GOLDMAN

(Communicated by S. I. Wawilow, Member of the Academy, 8. XI. 1937)

It is a well-known fact that an increase in the pressure of a gas causes its electric strength to increase(1).

The object of this investigation was to ascertain the limit up to which the electric strength of a gas may be raised by increasing its pressure. A high enough field voltage can evidently result in a direct tearing-out of electrons from the metal. This would naturally alter the nature of the dependence of breakdown voltage on pressure.

Experimental Arrangement and Conditions

All the measurements were made in the apparatus used in previous experiments. Lest the gap between the electrodes should be affected by the deformation of the apparatus due to compressed gas, the electrodes were mounted on a special standard (Fig. 1), only one end of which was screwed into the electrode, which was rigidly connected with the body of the apparatus. The other electrode of the standard was connected with the electrode of the apparatus by a flexible conducting wire. The distance between the electrodes was adjusted with a specially calibrated formers, the contact being verified with a galvanometer. The electrodes used were a steel sphere 20 mm in diameter, and a flat disc 50 mm in diameter. Discs made of different materials were used.

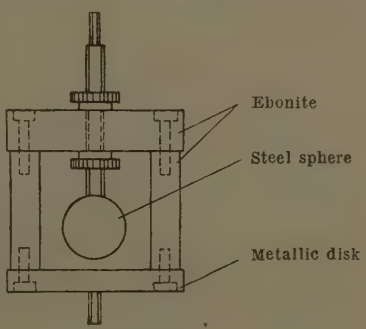


Fig. 1. Arrangement of electrodes.

The compressed nitrogen was supplied by the «Autogene» plant No 1.

In order to free the gas from impurities, which lower considerably the breakdown voltages, more especially if the gap is small, the gas was passed through two glass filtres before reaching the apparatus.

The nitrogen used contained 0.2—0.5% oxygen. No influence of variations in the O_2 content within this range has been detected.

Measurement Results

All the measurements were made at a 50 H alternating voltage. The distance between the electrodes varied from 50 μ to 1 mm. Such breakdown voltage values as were obtained immediately upon filling the apparatus with fresh gas were invariably lower than the subsequent ones. Further values were higher and showed little subsequent variation. The fact that,

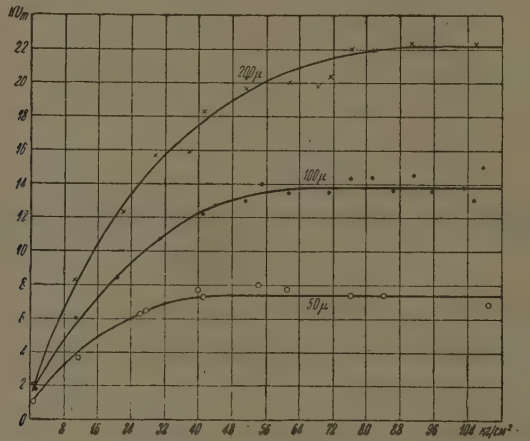


Fig. 2. The dependence of breakdown voltage of nitrogen on pressure for various distances between the electrodes.

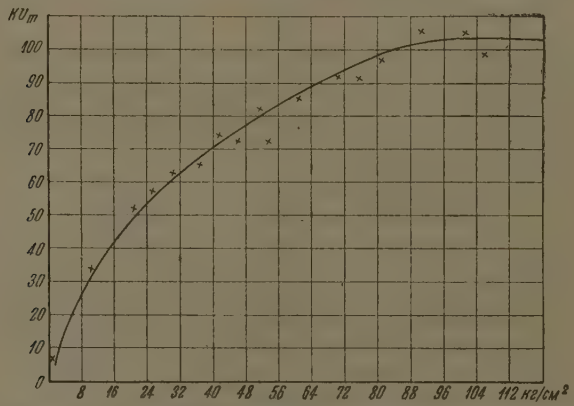


Fig. 3. The dependence of breakdown voltage of nitrogen on pressure. Distance between the electrodes is 1 mm.

after the first few punctures of a fresh gas, the breakdown voltages increased, would appear to indicate that the gas was at first being freed from impurities, which had got into the apparatus, in spite of the two filters placed in the path of the gas.

No value differed from the average by more than 10%, and for small gaps only did the difference amount to as much as 15%.

With small distances between the electrodes, especially when they were only 50 μ, the value of the breakdown voltage was to a great extent depen-

dent on the purity of the electrodes. With a 50 μ gap, the electrodes had to be cleaned after a series of 15—20 punctures. Only then was it possible to obtain reproducible values of breakdown voltages.

Every point plotted on the curve is an average of a few scores of punctures.

Figs 2 and 3 show the dependences of breakdown voltages on pressure, for various distances between the electrodes. As may be seen from these curves, when pressure has increased to a certain point, the breakdown voltages do no longer increase with an increase of pressure, whatever the spark gap (between 50 μ and 1 mm).

The pressure at which this phenomenon (saturation) occurs depends on the distance between the electrodes. The greater the spark gap, the higher the pressure required to produce saturation. For 50 μ, saturation sets in at 40 kg/cm², and for 1 mm, at 96 kg/cm². A change in the distance between the electrodes alters the field voltage at which saturation sets in. Fig. 4 shows

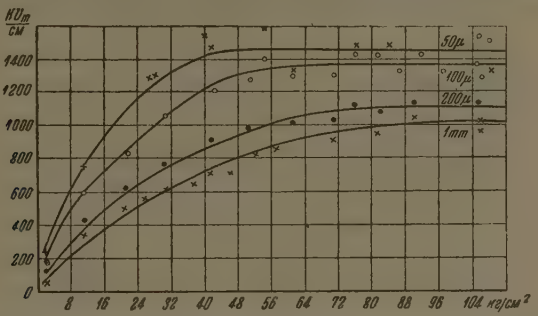


Fig. 4. The dependence of breakdown voltage of nitrogen on pressure for various distances between the electrodes.

the dependence of breakdown voltages on pressure for the same electrode spacings as shown in Figs 2, 3.

As may be seen from this figure, the breakdown voltage decreases with the increase of the spark gap. The voltage at which saturation occurs is also lowered. Thus, for 50 μ, saturation occurs at $E = 1450 \frac{\text{kV}_m}{\text{cm}}$, and, for 1 mm, at $E = 1000 \frac{\text{kV}_m}{\text{cm}}$.

All the above experiments were made with a steel sphere and a flat brass disc as electrodes.

To determine the influence of the electrode material breakdown voltages were also measured in using a steel sphere and a flat aluminium disc as electrodes. The aluminium used was chemically pure, the spark gap being 100 μ. The breakdown voltages proved to be 8—10% higher than those obtained with a flat brass disc.

Conclusions

The presence of saturation for all the breakdown voltages investigated shows that, starting from a certain pressure, the mechanism of gas puncture is altered. Impact ionization no longer plays a predominant part. With those fields as are obtainable at high voltages, electrons may be torn out from the cathode by the action of the field.

The field voltage required to produce a breakdown depends on the distance between the electrodes and varies from $1000 \frac{\text{kV}_m}{\text{cm}}$ to $1480 \frac{\text{kV}_m}{\text{cm}}$ for

distances from 1 mm to 50 μ. In this case, the voltage was calculated in the same way as for a steady field. If account is taken of the correction for the deformation of the field when sphere—disc electrodes were used, one should decrease E by 4% for the greatest distance of 1 mm.

These voltages determine the limit of electric strength for compressed nitrogen. The critical value of the field voltage at which the breakdown begins, depends on the distance between the electrodes. Since it is hardly likely that the process of the tearing-out of the electrons from the metal should depend on the distances between the electrodes, it is to be assumed that the breakdown is determined not only by the critical value of the field, but also by the mechanism of the subsequent motion of the electrons towards the anode.

No practical advantage would be gained by increasing the voltage to a maximum, since the breakdown voltages, while increasing very rapidly with an increase of pressure, ultimately increase at a steadily falling rate.

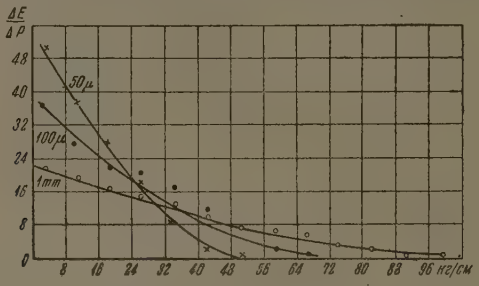


Fig. 5. The intensity of the increase of voltage with increasing pressure for various distances between the electrodes.

This is most apparent with high breakdown voltages. With a spark gap of 1 mm, a fourfold increase of pressure, say, from 24 kg/cm² to 96 kg/cm² results in an increase of breakdown voltages from $500\frac{kV_m}{cm}$ to $1000\frac{kV_m}{cm}$, i. e. a 100% increase. If, however, pressure is increased from 48 kg/cm² to 96 kg/cm² this increases the critical values of the field by 25%. Fig. 5 shows the dependence of the intensity of the increase of voltage on pressure. This dependence is of an exponential nature. For small distances $\frac{\Delta E}{\Delta P} = f(P)$ varies more sharply than for greater ones, the former being affected by pressure to a greater extent then the latter.

In testing the electric strength of the vacuum, Anderson [2] obtained, for a 1 mm gap, a critical value of the field equal to about $700\frac{kV_m}{cm}$. According to his data an increase in the gap caused a sharp fall in the critical voltage. A comparison of pressure and vacuum as dielectrics is thus seen to show the advantage of the former.

In point of electric strength, a compressed gas is comparable to insulating paper impregnated with transformer oil, which, in thicknesses of about 1 mm, has a breakdown voltage of $1000\frac{kV_m}{cm}$.

Physical Institute.
Academy of Sciences of the USSR.

Received
14.XI.1937.

REFERENCES

[1] Reher, Arch. f. El., XXV, H. 4 (1931); Zeier, Ann. d. Ph., **14,4,** 415 (1932); Б. М. Вул и И. М. Гольдман, Журн. техн. физ., VI, 8, 1613 (1934); VI, 2, 244 (1936). [2] H. W. Anderson, The Rev. of Scient. Inst., **6**, 309 (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

PHYSICS

THE SPLITTING-UP OF THE RAYLEIGH LINE AND THE ACOUSTIC SPECTRUM OF CRYSTALS

By E. GROSS

(Communicated by D. S. Rožděstvenskij, Member of the Academy, 14. XI. 1937)

As shown by the writer's former experiments (1), the scattering of light by liquids causes the monochromatic spectral line of the exciting radiation to be transformed into a triplet* (one unmodified line, and two modified ones**). The two modified components of the triplet are produced by Debye's longitudinal heat waves, and have been predicted by the theories of Brillouin (2) and Mandelstam (3). Landau and Placzek (4) accounted for the undisplaced component of the triplet. According to their theory, the ratio of the intensity of the undisplaced component to the total intensity of both displaced components should be $\frac{C_p - C_v}{C_v}$ where C_p and C_v are specific heats at constant pressure and volume. The ratio of the intensities of the components of the triplet as observed experimentally (5) is of the order required by that theory.

I have already communicated (6) some preliminary results of my observations on the splitting-up of the Rayleigh line in crystals (quartz). Crystals show a greater number of displaced components than provided for by the theories of Brillouin and Mandelstam. Experiments with crystals are most difficult because of the extreme faintness of the scattered light. Recent researches with crystalline quartz warrant the conclusion that the Rayleigh line of crystals appears to have six modified components, i. e. three components on each side of the undisplaced line (seven components in all, including the undisplaced one***).

I believe that this discrepancy between the experiment and the theory, which predicts a doublet in scattered light also for crystals, may be accounted for by the elastic properties of solids.

The acoustic spectrum of a crystal is known to consist of three parts Along any given direction within the crystal, three different kinds of elastic heat vibrations (a longitudinal vibration and two kinds of transverse ones) can spread with different velocities of propagation. As in the case of liquids, the longitudinal heat waves give rise to two modified components in the

* If the details of the phenomenon are neglected.

** Cf., for instance, the microphotometric record taken from the spectrogram of the triplet, obtained by the writer with a Michelson echelon grating and reproduced in my notice in Nature, **129**, 722 (1932).

*** The problem of the existence of an undisplaced component and its intensity is difficult to solve. The light scattered by the crystals being very faint, a trace of parasitic light suffices to distort the results of the experiment. The intensity of the undisplaced component, if the latter does exist, is in any case not higher (apparently lower) than that of the displaced components.

spectrum of scattered light of the crystal. The fact that the Rayleigh line in crystals has four additional modified components would, in my opinion, be accounted for by the scattering of light by two kinds of transverse heat waves, which the Brillouin-Mandelstam theory leaves out of account. Like the variations of density of longitudinal waves, the shearing strains, distributed periodically along the transverse heat wave trains, cause local variations of the index of refraction. Such unhomogeneities in the optical properties of solid media must produce the scattering of light. As in the case of longitudinal heat waves, the scattering of light may be interpreted as the reflection of light from the spatial lattices formed by the transverse heat wave trains. Only discrete components of the Rayleigh line (two doublets) can therefore arise due to interference in the spectrum of the light scattered by transverse heat waves.

In amorphous solids, where the velocity of transverse waves is identical for all directions of vibrations, only one doublet due to transverse waves should be observable. In amorphous solids, therefore, the Rayleigh line should have in all four modified components, including the doublet due to longitudinal waves. It is difficult to verify this experimentally. The undisplaced line of amorphous solids must have, owing to the great «stationary» unhomogeneities of their structure, a much greater intensity than the displaced components. Being predominant in the spectrum, it may obscure them during examination, and this is what actually happens (7).

These simple considerations suggest the same conclusions as to the structure of the Rayleigh line in solids as does the detailed theory of the phenomenon given by Placzek and Landau*.

It is but natural that only one doublet, that is due to longitudinal waves, is observed in ordinary liquids, since no transverse vibrations are present in a liquid**.

The above considerations as to the structure of the Rayleight line in solids are confirmed by some of the latest results of optical research of ultra-sonic vibrations. The investigation of light diffraction by ultra-sonic waves in solids (glass) has recently brought to light (8) some additional diffraction maxima. As shown by research, this phenomenon may be attributed to the diffraction of light by gratings created by ultra-short transverse waves arising simultaneously with longitudinal ultra-sonic waves due to the oscillations of a piezo-electric quartz crystal. The diffraction of light by ultra-sonic waves and the scattering of light by Debye's elastic heat waves are known to be phenomena of the same nature. The existence of the diffraction of light by transverse waves is, consequently, in good agreement with the appearance of the additional modified components of the Rayleigh line in crystals and confirms their interpretation as developed in this communication.

Physical Institute.
Leningrad University.

Received
1.XII.1937.

REFERENCES

[1] E. Gross, Nature, **126,** 201 (1930); Nature, **129,** 722 (1932). [2] L. Brillouin, Annales de Phys., **17,** 88 (1922). [3] L. Mandelstam, J. Russ. Phys. Chem. Soc., **58,** 831 (1926). [4] L. Landau u. G. Plaszek, Phys. ZS. d. Sowjetunion, **5,** 172 (1934). [5] E. Gross, Nature, **126,** 400 (1930); K. Birus, Phys. ZS., **37,** 548 (1936). [6] E. Gross, ZS. f. Phys., **63,** 685 (1930). [7] E. Gross, ZS. f. Phys., **63,** 685 (1930); Nature, **129,** 722 (1932). [8] E. Heidemann u. K. H. Hoesch, Naturwiss., **23,** 705 (1935).

* As yet unpublished. I am very much indebted to G. Placzek, and also to L. D. Landau, for their interesting dicussion of the consideration set forth above, and for the communication of the results of their as yet unpublished work.

** If the possibility of the existence of transverse vibrations at high frequencies in ordinary liquids is neglected.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

CHEMIE

ÜBER DAS GESETZ DER PERIODIZITÄT

Von P. PETRENKO-KRITSCHENKO, korrespondierendem Mitglied der Akademie

In vorliegender Notiz will ich das Fazit meiner Arbeit ziehen, deren Ergebnisse bereits im Jahre 1922 den Gegenstand einer Abhandlung bildeten (1). Zeitlich umfasst die Arbeit einen bedeutenden Abschnitt des zwanzigjährigen Bestehens der Sowjetmacht.

Mich beschäftigt stets die Frage, ob die Veränderung der Eigenschaften bei Anhäufungen unstetig vor sich geht, hierbei das Gesetz der Periodizität oder Teile desselben, die einzelnen Knicke—Gesetz des Übergangs der Quantität in die Qualität—befolgend, oder aber kontinuierlich, an Engel's Bemerkung erinnernd: «Man bekommt eine Menge von Veränderungen der Eigenschaft zu beobachten, hinsichtlich deren durchaus nicht nachgewiesen ist, dass sie durch quantitative Veränderungen bedingt sind».

Nachstehend eine Orientierungstabelle der von mir untersuchten Aktivitätskurven der Halogenverbindungen des Methans.

An diesen Kurven werden beide Arten von Regelmässigkeiten beobachtet. Die zwei ersten Kurven sind von entgegengesetztem periodischem Charakter: die eine ist gewissermassen das Spiegelbild der andern. Ein Teil der Kurven für Mono-, Di- und Trisubstituierten hat einen konstanten Charakter. Die Kurven verhalten sich gleichartig sowohl gegen Chlor-, Brom- als auch gegen Jodverbindungen*. Inkonstant ist der Teil der Kurven, der die tetrasubstituierten Verbindungen kennzeichnet. Die Kurven für Brom- und Jodverbindungen liegen, entsprechend der grösseren Aktivität dieser Verbindungen, höher als für Chlorverbindungen. Hier lässt sich eine Tendenz zum Verschwinden der Knicke erkennen. Noch höher ist die Aktivität für Reaktionen bei Beteiligung von Kolloidsilber und Silbernitrat. Die Kurven sind besonders hoch für Verbindungen von gemischter Zusammensetzung.

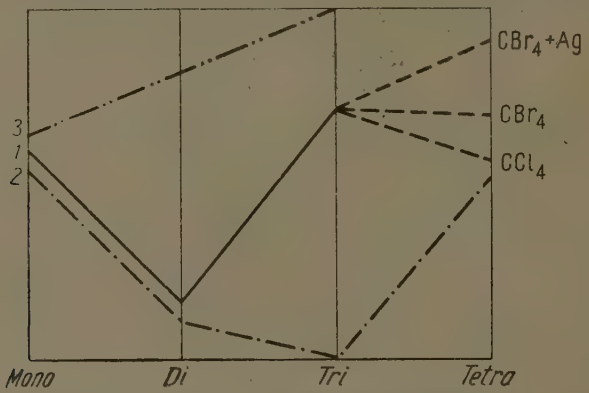
Tabelle für Ag und $AgNO_3$

Temp. °C	Zeit in Std.	Konzentration				
30	24	$n/20$	Cu_3Cl 13	CH_2Cl_2 0	$CHCl_3$ 12	CCl_4 12
40	3	$n/20$	CH_3Br 10	CH_2Br_2 0	$CHBr_3$ >90	CBr_4 100
13	24	$n/20$	CH_3S 60	CH_2S_2 20	CHS_3 >90	— —
20	3	$n/20$	$C_6H_5CH_2Cl$ 6	$C_6H_5HCl_2$ 55	$C_6H_5CCl_3$ 62	— —
20	50	$n/20$	$CH_2ClCOOH$ 1.5	$CHCl_2COOH$ 3.5	CCl_3COOH 27.5	— —
25	1	$n/50$	$CH_2BrCOOH$ 0	$CHBr_2COOH$ 9	CBr_3COOH 70	— —

* In dieser Abbildung sind die Kurven der Jodverbindungen nicht zur Darstellung gebracht.

Diese und andere in eingehenden Aufsätzen von mir verzeichneten Tatsachen sprechen für das Verschwinden der Periodizität: die Relationen gehen in das Gebiet der hoch verlaufenden Kurve *3* über.

Eine ebensolche stetig ansteigende Kurve für Chlorverbindungen des Methans ist von Polanyi erhalten worden. Er untersuchte die Wechselwirkung mit Natriumdämpfen. In meinen Versuchen wurden die Messungen nicht bei einer so hohen Temperatur und nicht mit einem so starken Reaktiv ausgeführt. Bei den Messungen, die in meinem Laboratorium vorgenommen wurden, kamen alkoholische Lösungen zur Verwendung, und die Messun-



gen erfolgten unterhalb von 100°: bei diesen Bedingungen macht sich die Periodizität geltend.

Hantzsch nimmt in der Polemik gegen mich anlässlich der gefärbten Verbindungen an (²), dass Polanyis Messungen mit meinen im Widerspruch stehen. Liegt hier aber tatsächlich ein Widerspruch vor? Hier haben wir keinen Widerspruch, sondern eine Erweiterung der Regelmässigkeit, die von mir für den Verlauf der Kurven der tetrasubstituierten Halogenderivate des Methans festgestellt worden ist: Mit der Zunahme der Aktivität beim Übergang von den Chlor- zu den Brom- und Jodverbindungen werden die Knicke weniger deutlich: Übergang zur Kontinuität.

Möglicherweise lässt sich diese Regelmässigkeit durch folgende Hypothese erklären. Die Langsamkeit der Wechselwirkung weist auf irgendeine innere Arbeit hin, die in Knicken der Kurven, in der Periodizität zum Ausdruck kommt. Die Zunahme der Geschwindigkeiten zeugt von einer Verminderung dieser Arbeit und daher von einer Tendenz zur Kontinuität.

Besagte Hypothese kann man durch einen Vergleich meiner Messungen mit Waldens Arbeit über die Umkehrung des optischen Vorzeichens stützen. KOH reagiert nach meinen Messungen langsam, die Silberreagenzien rascher. Dementsprechend haben wir bei Walden im ersten Falle eine Umkehrung und im zweiten die Erhaltung des optischen Vorzeichens.

Eingegangen
d. 29. X. 1937.

ZITIERTE LITERATUR

[1] ЖРФО, 54, 447 (1929); 306, 320 (1927); 149 (1928); 1777 (1929); 177 (1930); ЖОХ, III, 581, 587; IV, 85. [2] Ber., 1049, 1771 (1933), 1349 (1934), 618 (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

AGROPHYSIOLOGY

RESISTANCE OF WINTER WHEATS TO BLACK BACTERIOSIS (*BACTERIUM TRANSLUCENS* VAR. *UNDULOSUM* S. J. R.)

By M. V. GORLENKO

(Communicated by A. A. Richter, Member of the Academy, 13. XI. 1937)

As early as in 1920, N. N. Voronichin, one of the leading Soviet mycologists wrote: «I have no doubt personally that only the study of the physiology of the diseased plant organism, as well as the investigation of the immunity phenomena in the plant world, will enable us to cope with the parasitic diseases in plants» (2).

The study of the immunity phenomena in plants was started in the USSR in 1911 by N. I. Vavilov(1), Member of the Academy. He studied in detail the susceptibility of cereals to rust and mildew. Since that time, numerous papers have dealt with the problem, and the susceptibility of cereals to these diseases, as well as to smut, has been sufficiently elucidated(5).

During the few last years, a number of new diseases has been discovered, and it is believed that these diseases should be combated chiefly by selecting resistant varieties. Black bacteriosis is the most important of these diseases.

The black bacteriosis of wheat, caused by *Bacterium translucens* var. *undulosum* S. J. R., was described in 1917 by E. Smith (7). The disease affects the wheat ears, leaves and stalks. In the USSR one has to deal chiefly with the blackening of the ears, which is known in the USA under the name of «black chaff». The disease has been studied in the USSR by Fomin and Gorlenko. The latter has studied the destructiveness of the disease, its distribution, the sources of infection and the methods of combating it (3, 4).

This is the fourth of a series of papers devoted to the investigation of wheat bacterioses.

In the present paper, data on the susceptibility of winter wheat races to black bacteriosis are set forth.

As early as in 1934, Fomin (6) wrote that the ear bacteriosis of the black chaff type attacks mainly the red-eared races (*alborubrum*, *milturum*, *ferrugineum*). We also succeeded in observing this, though with regard to a comparatively small number of varieties.

In 1937, we had the opportunity of observing the disease in a great number of winter wheat varieties on the plots of the Kamenno-Stepnaya Experiment Station.

The result of our observations show quite clearly that black bacteriosis attacks definite races of winter wheat. In all, we analysed 68 varieties on the «steppe» plot and 250 varieties on the «geographical» plot.

The results are shown in Tables 1 and 2.

Table 1

The Susceptibility of Different Winter Wheat Races to Black Bacteriosis on the «Steppe» Plot, Talovaya, 1937.

Winter wheat races	Number of varieties examined	Number of varieties in which diseased ears were found	
		Absolute number	Percentage
Ferrugineum	16	11	68.7
Milturum	4	3	75.0
Lutescens	8	1	12.5
Erythrospermum	30	9	30.0
Velutinum	2	0	0
Albidum	1	0	0
Hostianum	3	1*	33.3
Caesium	1	0	0
Nigroaristatum	1	0	0
Barbarossa	2	0	0

Table 2

The Susceptibility of Different Races of Winter Wheat *(Tr. vulgare)* to Black Bacteriosis on the «Geographical» Plot, Talovaya, 1937.

Winter wheat races	Number of varieties examined	Number of varieties in which diseased ears were found	
		Absolute number	Percentage
Ferrugineum	31	14	43.3
Milturum	21	9	47.6
Alborubrum	2	2	100.0
Erythrospermum	105	2	0.9
Lutescens	41	1	2.4
Albidum	12	0	0
Caesium	2	0	0
Hostianum	1	0	0
Nigroaristatum	4	0	0

It should be added that varieties of *lutescens*, *erythrospermum* and *hostianum* usually have an insignificant percentage of diseased ears. In *erythrospermum* we found only one variety having 30% of diseased ears, in *lutescens* also one (017) having 28%, in *hostianum* all the diseased varieties showed only traces of the disease (single diseased ears).

The data tabulated above show that the varieties of *alborubrum*, *milturum* and *ferrugineum* (i. e. the red-eared), are the oftenest and the most strongly infected by the disease, independenly of wheather they are awned or awnless. We refer these races to the first group.

The varieties of the races *erythrospermum* and *lutescens* occupy an intermediate position. Strongly infected varieties are very rare among them. The percentage of infected varieties is also low. These races constitute the second group.

* In one of the varieties two or three diseased ears were found.

To the third group we refer the varieties wherein no high percentage of diseased ears was observed or no diseased ears were found at all. Here belong the races *albidum*, *velutinum*, *hostianum*, *nigroaristatum*, *barbarossa*, i. e. chiefly the white-eared forms.

Thus we can draw up the following tentative classification of races of winter wheat (*Tr. vulgare*) with regard to their susceptibility to black bacteriosis (Table 3).

Table 3

Strongly susceptible races	Races occupying intermediate position	Resistant races
Alborubrum	*Erythrospermum*	*Velutinum*
Milturum	*Lutescens*	*Hostianum*
Ferrugineum		*Nigroaristatum*
		Barbarossa
		Albidum

Such a marked difference in the susceptibility of races whose ears differ in colour, may be probably due to the chemical composition of the sap, or rather of the pigment responsible for the ear colour.

The solution of this problem may be approached by cultivating the bacteria causing the disease in extracts of the ears of resistant and non-resistant races. Should the growth of the bacteria in extracts of the resistant races prove weak, we will be justified in believing the resistance of the different races to be due to their chemical peculiarities. This will enable us to establish the nature of the immunity of wheat to black bacteriosis.

The marked difference in the susceptibility of the different winter wheat races to black bacteriosis permits us to hope that it will be quite possible to evolve a variety resistant to black bacteriosis*, as well as to other diseases, even within the races susceptible to the disease in question. We are of the opinion that, in selecting resistant varieties, the susceptibility to black bacteriosis should be taken into consideration.

Laboratory of Phytopathology.
Plant Protection Station.
Voronezh.

Received
11.XI.1937.

REFERENCES

[1] Н. И. Вавилов, Тр. Селекц. ст. при Московск. с-х. ин-те, 1—110 (1913). [2] Н. Н. Воронихин, Грибные и бактериальные болезни с.-х. растений, стр. 1 (1922). [3] М. В. Горленко, Защита растен., № 3, 82—85 (1935). [4] М. В. Горленко, Защита растен., № 8, 109—114 (1936). [5] Л. Ф. Русаков, Всесоюзная конференция по ржавчине. Тезисы докладов, ВАСХНИЛ, стр. 15—24 (1937). [6] Е. Е. Фомин, Тр. лаборатории фитопатологии УНИЗР, № 1, 88—133 (bacteriosis on p. 122). [7] E. Smith, Journ. of Agric. Res., X, № 1, 51—54 (1917).

* Let it be remembered that we mean the disease of ears, and not of the vegetative organs. The latter is extremely rare in this country. In Western Europe, the disease of the leaves is likewise rare.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume, XVIII, № 2

PLANT PHYSIOLOGY

ON THE EXCRETION OF NITROGENOUS COMPOUNDS BY THE ROOTS OF LEGUMINOUS PLANTS (LUPINE) AND THE CONVERSION OF THESE COMPOUNDS IN THE PRESENCE OF BACTERIA

By A. A. ISAKOVA and V. A. ANDREJEV

(Communicated by A. A. Richter, Member of the Academy, 17. XI. 1937)

The fact that the plant roots excrete various organic and inorganic compounds may be regarded as proven. As shown by Virtanen's investigations, the root system of leguminous plants when inoculated with nodule bacteria, excrete into the surrounding medium nitrogenous compounds of the amino-acid type. Virtanen believes that the excretion of amino-compounds by leguminous plants is due to the vital functions of nodule bacteria and refers these substances to the asparaginic acid type.

In experiments with mixed cultures of leguminous and cereal plants, the author succeeded in proving that it is possible for the cereals to utilize the nitrogenous compounds excreted by the leguminous plants. However these results found no confirmation in the experiments of Romašev and Ludwig.

In 1937, we made experiments with a view to ascertaining whether the roots of leguminous plants (viz. lupine) excrete nitrogenous compounds, and determining the conversions of these compounds in the presence of various bacteria. The experiments were carried out in vessels for sterile cultures specially designed for the purpose, the blue lupine being the plant under experiment. Each vessel contained one plant. There were five series of the experimental variants and twelve of the control ones. The sterility of the cultures was tested by introducing a drop of the nutritive solution taken from the vessel into a peptone broth. The lupine seeds were sterilized for half an hour in a 1% bromine solution. The seeds were planted on February 9, 1937.

The plan of the experiment was as follows:

1) Control, sterile cultures.
2) Inoculation with azotobacter.
3) » » lupine nodule bacteria.
4) » » » bacteriorhiza.
5) » » soil solution.

The inoculation was effected by means of introducing 1 cm³ of the bacterial suspension into the vessel before planting the seeds.

The inoculation with azotobacter and nodule bacteria was effected by washing a test-tube of agar culture with sterilized tap water to the amount of 50 cm³, and collecting the water into a sterile flask. In inoculating with

lupine bacteriorhiza 2 g of fresh roots, freed from the adhering lumps of the soil, were introduced into a sterile flask, sterilized tap water being then poured over them. After stirring for one minute, the suspension was ready for use. For inoculating with soil solution, 1 g of hothouse soil was used; the rest of the procedure was as in the preceding cases.

In filling the vessels with sand, the latter was soaked in Prianišnikov's nutritive solution. The vessels were first sterilized. In the case of azotobacter and nodule bacteria, the nutritive solution contained no nitrogen. The plants were watered with sterile Prianišnikov's nutritive solution of a weaker concentration (half as strong as before). Analyses of the root excretions were made in samples obtained by washing the sand with the same nutritive solution. The design of the vessels ensured the sterility of the samples. The possibility of anaerobiosis was precluded by air circulation. Air passed through the sand continuously. The samples were taken twice, on March 22, 1937, after one and a half month's development of the plants, and on May 3, after a three-month culture. The samples were tested for the presence of NH_3 and amino-nitrogen in the medium. The ammonia content was determined after the Lange method; the amino-acids, after van Slyke method.

The results obtained are tabulated below. The content of NH_3 and amino-groups is expressed in terms of milligrams per one plant.

Content of NH_3 and Amino-Nitrogen in the Medium at Different Dates

Variants of the experiment	Content of NH_3 in mg per plant			Content of amino-nitrogen in mg per plant		
	March 22	May 7	Total	March 22	May 7	Total
Control	—	2.206	2.206	7.32	—	7.32
Azotobacter	0.151	0.246	0.397	4.41	7.94	12.45
Nodule bacteria	—	0.177	0.177	1.56	—	1.56
Lupine bacteriorhiza	9.445	—	9.445	4.29	12.07	16.36
Soil solution	6.538	0.266	6.804	3.47	1.42	4.89

The figures of the table show that an accumulation of NH_3 in the medium takes place in all the variants. The causes of this accumulation, however, are different in the different cases.

In the control vessels, the presence of NH_3 is due to the composition of the solution and not to its being excreted by the root system, inasmuch as the root excretions were washed with Prianišnikov's nutritive solution, which contains NH_4NO_3.

The variants where plants were inoculated with azotobacter and nodule bacteria also showed an accumulation of NH_3 in the medium, and this accumulation should be attributed to the assimilatory activity of the organisms in question, since there was no nitrogen in the nutritive solution.

The accumulation of NH_3 in the lupine bacteriorhiza and soil solution variants may be due to the development of ammonifiers and denitrifiers, which inhabit the lupine bacteriorhiza in great numbers. This assumption was corroborated by inoculations of different media with bacteria taken from the lupine bacteriorhiza. In these experiments the greatest number of surviving colonies was observed on media containing peptone and complex nitrogen compounds.

The excretion of complex nitrogen compounds by the lupine roots, when cultivated under strictly sterile conditions, is considerable. It is observed

to occur in large quantities when the plant is young; at later stages of the plant's development, it decreases or ceases altogether.

The variant where the plant was inoculated with azotobacter shows, at the first stage of the plant's development, a certain drop in the accumulation of the amino-nitrogen, the amino-groups being by all probability intercepted by the azotobacter itself. At the time of taking the second sample, an increased amino-nitrogen content in the medium was recorded, the exosmosis in the control having completely stopped. It is to be assumed that such a change in the accumulation of amino-acids is due to a derangement of the metabolism in the azotobacter, because the lupine root excretions do not set up conditions favourable to the development of this organism.

The inoculation of the plant with nodule bacteria resulted in a sharp decrease of the amino-nitrogen content in the medium during the first stages of the plant's development. This is at variance with Virtanen's results. The bacterial cells seem, under the conditions of our experiments, to intercept the amino-nitrogen. When the bacteria had penetrated into the root tissues and nodules had formed in great numbers, the excretion of amino-acids by the roots ceased.

The variant where the plant was inoculated with the lupine bacteriorhiza shows the highest accumulation of amino-nitrogen in the medium. It may be associated with the development of a specific microflora and the metabolism of the microorganisms inhabiting the bacteriorhiza.

The inoculation of the plant with the soil solution causes a drop in the accumulation of amino-nitrogen, as compared with the control.

On the basis of what has been set forth above, we are justified in drawing the following conclusions:

1) In sterile sand cultures of lupine, amino-nitrogen is observed to be excreted in considerable quantities. The process is more vigorous when the plant is young.

2) The inoculation of lupine with azotobacter results, during the first stages, in a decrease of the content of amino-groups in the medium, but subsequently they are accumulated in great quantities. In the latter case, the decomposition of the protein of the azotobacter cells seems to take place.

3) The inoculation with nodule bacteria during the first stages of the plant's development results in a sharp decrease of the amino-nitrogen content in the medium, the amino-nitrogen being untercepted by the bacterial cells. Later on, the exosmosis of amino-acids ceases altogether.

4) The inoculation of the plant with bacteriorhizal organisms increases the accumulation of amino-acids in the medium. These acids may be associated with the metabolism in bacteria.

Received
17.XI.1937.

REFERENCES

[1] A. Virtanen, Bioch. ZS., **258**, 1—4, 106—117 (1933). [2] A. Virtanen u. S. von Hausen, Bioch. ZS., **232**, 1 (1931). [3] A. Virtanen a. S. von Hausen., J. Agr. Sci., **25**, 278—289 (1935). [4] S. A. Ludwig a. E. A. Franklin, Bot. Gazette, **98**, № 4, 680 (1937). [5] А. Ромашев, Химиз. соц. земл., № 11 (1936).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

PLANT PHYSIOLOGY

ON THE PROBLEM OF PHOTOPERIODICAL AND TEMPERATURE AFTER-EFFECTS

By M. ABAKUMOVA

(Communicated by N. I. Vavilov, Member of the Academy, 25. XI. 1937)

A number of investigations[Garner and Allard (7), Rasumoff (1,5) and others] in their papers on photoperiodism pointed out that there is a close relation between the growth and development of plants and length of the day. The length of the day as one of the factors of environment, acquires particular significance in research work on problems of ontogenetical development, in connection with the theory of stage development of plants worked out by T. G. Lysenko (4), of the Academy.

Furthermore many investigators in this field [Eguis (6), A. Volk (8) and others] point out facts of photoperiodical after-effects, which appear not immediately, but in subsequent generations. This after-effects, as the authors note, consists in the shortening or lengthening of the vegetative period of the progeny. So, Volk states that the plants of the soy-bean grown from seeds which ripened submitted to conditions of a long day, flowered 14 days later and produced ripe seeds 3 weeks later than those plants, which were grown from seeds ripened in short day conditions. Analogous data are presented by Eguis.

The experimental results of I. A. Kostučenko and T. J. Zarubailo (2, 3) enabled them to conclude that the embryo formed, but not having reached the dormant period, i. e. the embryo of a ripening, but not yet ripe seed, may, if submitted to suitable temperature conditions, pass through the stage of yarovization, just as the embryo of a ripe seed which began to germinate.

Hence, taking into account that the soy-bean plant is a «short-day» plant and that it requires relatively high temperatures to pass through the stage of yarovization, one may assume that the «photoperiodical after-effect» observed in the experiments of Eguis and Volk should be put down to the account of different temperature conditions in which the forming and ripening seeds of the darkened plants were placed, in comparison with the control ones. The sole significance of the difference in the length of day consisted in the fact that the control plants which grew on normal day, flowered and ripened later and, consequently, were submitted to lower autumn temperatures, than the plants grown on short day.

To verify this assumption we conducted a series of experiments on wheat in 1936 and 1937 (town Puškin, near Leningrad).

In 1936 we made several sowings of different varieties of winter and summer wheat on a photoperiodical plot. Plants of wheat in one case were

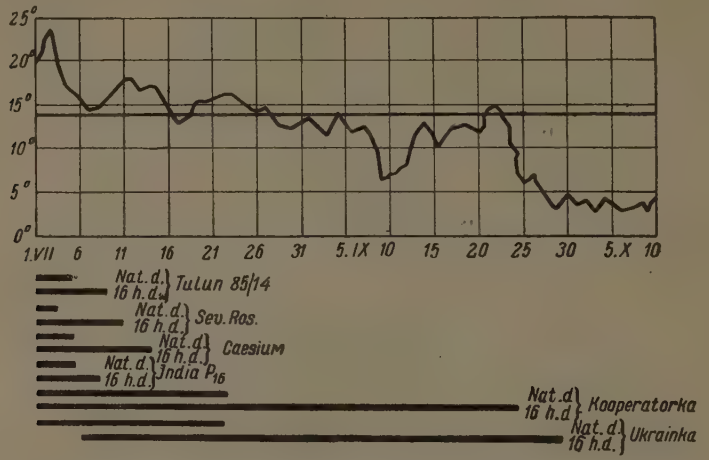


Fig. 1. Chart of mean 24-hour temperatures, August—September 1936, town Puškin, and data of ripening of wheats on photoperiodical plot.

grown submitted to natural day conditions of 18—20 hours, and in another, were grown under a 16-hour day, summer varieties being sown out without

Table 1

Data on Ear-Formation and Ripening of Summer and Winter Wheat Grown at Different Photoperiods (town Puškin, 1936)

Name of variety	Natural day, 18—20 hours		16-hour day		Retardation of ear-formation in plants grown on short day in comparison with plants grown under natural day (days)
	Date of ear-formation	Date of ripening	Date of ear-formation	Date of ripening	
Summer wheat:					
Tulun 85/14 var. *ferrugineum*	5. VII	5. VIII	8. VII	9. VIII	3
Northern *Rossicum* 0324 var. *ferrugineum*	4. VII	3. VIII	10. VII	11. VIII	6
Caesium 0111	3. VII	5. VIII	10. VII	14. VIII	7
24386 India P 16 var. *hostianum*	1. VII	5. VIII	2. VII	8. VIII	1
Winter wheat:					
Kooperatorka var. *erythrospermum*	9. VII	22. VIII	20. VII	24. IX	11
Ukrainka var. *erythrospermum*	11. VII	22. VIII	6. VIII	29. IX	25

previous yarovization, while the winter varieties were yarovized by keeping them at a temperature of 2—5° for 45 days. The sowing was made on 15. V.

The data given in Table 1 on the vegetative periods of summer and winter wheat grown at different photoperiods, show that the ripening of the summer variety of wheat, grown under natural day, was completed on August 5, while the ripening of that grown under a 16-hour day was completed on August 14. Referring to the chart of average day and night temperatures, taking in the period from ear-formation to ripening (Fig. 1), we see that ripening of summer varieties of wheat, grown under normal day as well as of those grown under a 16-hour day, took place at temperature higher than 15° C. The ripening of the winter varieties of wheat, grown under natural day, was completed on August 22, and likewise took place at a temperature of more than 15° C, that is in conditions analogous with the summer varieties. And, lastly, the plants of the same varieties of winter wheat, but grown under a 16-hour day, ripened September 24—29, a considerable part of the period of ripening having taken place at reduced temperatures (lower than 15° C).

The data of Table 1 further confirm the fact that the plants of winter varieties possess a longer light stage of development, than those of summer varieties.

Seeds of the wheat crop gathered in 1936, ripened at different photoperiods, were sown on plots on 13.V.1937 under natural day conditions. The summer varieties were sown without previous yarovization, the winter varieties were submitted to a temperature 2—5° C for 20 days.

Table 2

The Influence of the Photoperiodism After-Effect on the Time of Ear-Formation in the Progeny of Summer and Winter Wheat (town Puškin 1936)

Name of variety	Date of ear-formation		Acceleration in ear-formation of plants grown from seeds of the 16-hour day crop of 1936 (days)
	Plants grown from seeds ripened under natural day conditions	Plants grown from seeds ripened under a 16-hour day	
Summer wheat:			
Tulun 85/14 var. *ferrugineum* . . .	30. VI	29. VI	+ 1
Northern *Rossicum* 0324 var. *ferrugineum*	1. VII	29. VI	+ 2
Caesium 0111	2. VII	2. VII	0
24386 India P 16 var. *hostianum* . .	29. VI	27. VI	+ 2
Winter wheat:			
Kooperatorka var. *erythrospermum* .	23. VII	8. VII	+13
Ukrainka var. *erythrospermum* . . .	27. VII	3. VII	+24

The data of phenological observations given in Table 2 demonstrate the fact that the difference in dates of ear-formation in the progeny of the summer

wheat, grown in 1936 at different photoperiods, amounts to 0—2 days, i. e. the plants grown from seeds obtained in 1936 under conditions of a shortened 16-hour day do not differ practically in the length of their vegetation period from those, grown from seeds obtained under natural day conditions. Entirely different is the behaviour of the progeny of plants of winter varieties of wheat grown in 1936 at different photoperiods. In this case we observe that the period up to the ear-formation stage is shorter in plants grown from seeds crop of 1936 under a shortened day, than in those grown from seeds of 1936 crop, but in natural day conditions. This difference amounts to 13 days in the variety Kooperatorka *erythrospermum* and to 24 days in the variety Ukrainka *erythrospermum* (Fig. 2).

Fig. 2. Winter wheat Ukrainka var. *erythrospermum*. Spring sowing of not fully yarovized (20 days yarovization) seeds. Left-hand side (2 vessels)— plants from seeds grown under natural (18—20-hour) day and ripened at high (more than 16° C) temperatures. Right-hand side— plants from seeds grown under a shortened (16-hour) day and ripened at lower temperatures.

The evidence examined above warrants us to assume that facts of «photoperiodical after-effects» in subsequent generations, though not as a rule, may frequently represent not the direct effect of the length of day on the maternal plant but the consequence of the fact that the forming and development of the embryo at different photoperiods takes place at different temperatures, owing to which, some of the plants experimented upon, may produce seeds yarovized in a greater or less degree. Plants, grown from such seeds are distinguished as may be expected by shortened vegetative period. in comparison with plants of the same variety, but grown from seeds not submitted to the effect of yarovizing temperatures at the time of ripening.

It is necessary to bear this circumstance in mind, in conducting experiments on photoperiodism, so as not to draw false conclusions.

All-Union Plant Breeding Institute
Pushkin.

Received
25.XI.1937.

REFERENCES

[1] А. В. Дорошенко и В. И. Разумов, Тр. по прикл. бот. и сел., XXII, вып. 1 (1929). [2] И. А. Костюченко и Т. Я. Зарубайло, Сел. и семенов., № 3 (1935). [3] И. А. Костюченко и Т. Я. Зарубайло, Сел. семенов., № 6 (1937). [4] Т. Д. Лысенко, Теоретические основы яровизации (1936). [5] В. И. Разумов, Тр. по прикл. бот., ген. и сел., XXIII, вып. II (1930). [6] С. А. Эгиз, Издание Детскосельской акклимат.ст., вып. 9 (1928). [7] Garner a. Allard, Journ. of Agr. Res., **23** (1923). [8] A. Volk, Der Forschungsdienst, I, H. 2, 112 (1936).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

PLANT ANATOMY

ON THE FLINTINESS AND FLOURINESS OF WHEAT ENDOSPERM

By V. G. ALEKSANDROV and O. G. ALEKSANDROVA

(Communicated by N. I. Vavilov, Member of the Academy, 16. XI. 1937)

The phenomena of flintiness and flouriness of wheat grain are, besides their great economical importance, of considerable theoretical interest. For this reason, the cause of these most peculiar properties of the wheat grains has been the subject of investigations since the latter half of the last century (1) and still remains a subject of study (2). However, it is not yet definitely known to what combination of structural characteristics the flintiness of the endosperm tissue, on the one hand, and its flouriness, on the other, are due. The study of these phenomena under the microscope has shown their complexity and likewise the fact that in different cases they are the result of different combinations of the structural elements of the contents of the endosperm cells.

The investigation of Aleksandrov and Aleksandrova (2) has shown that the most important factor in the origination of flintiness and flouriness is the fine-grained starch, which they have called chondriosomal, as distinguished from the coarse-granulated plastid starch. So, for instance, when the granules of chondriosomal starch are angular and closely packed together, without considerable protein intercalations, then the structure is floury. But when the chondriosomal starch granules are round and the interstices between them, occupied by protein, are larger, then the structure is flinty. Observations show, moreover, that differences in the shape of the chondriosomal starch granules, correspond with differences in the colloidal and physical (optical) properties of the starch itself and also in the properties of the contiguous protein. The protein around angular starch granules is viscous, while that associated with round starch granules is elastic.

Our further investigations of the structural peculiarities of flinty and floury varieties of wheat endosperm have enlarged our understanding of flintiness and flouriness.

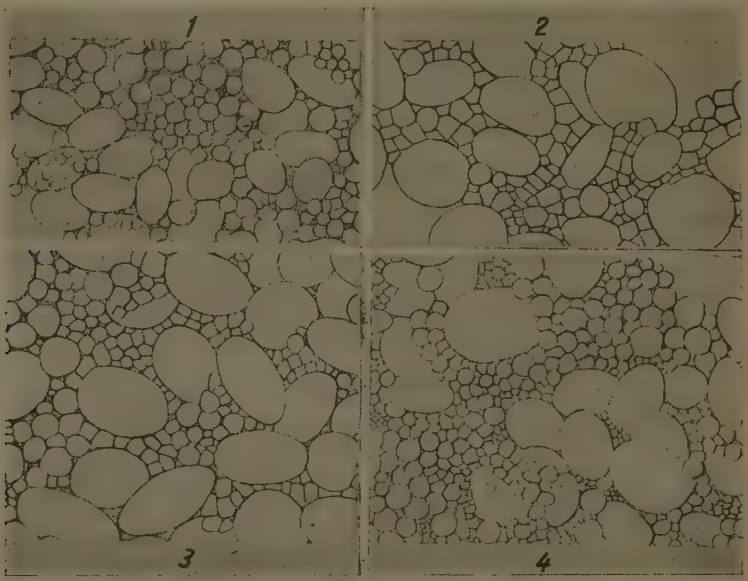
In this paper the results of our investigation are set forth. Of the material examined by us, we shall give only a few drawings. Fragments *1*, *2* and *3* (on the figure) show different parts of the endosperm tissue of an emmer of German origin [Tr. *dicoccum* (Schrank) Schübl. gr. *farrum* Bayle], while fragment *4* shows a section of a soft summer wheat from China (*Tr. vulgare* Host. *gr. lutescens* Al.). The magnification is everywhere the same. Fragment *1* is a case of a flinty structure of the endosperm tissue of emmer. Among the relatively small plastid starch granules are scattered chondriosomal starch

granules of two sizes, namely very fine ones and somewhat larger ones, forming, so to say, a transition to the plastid starch grains.

The main mass of the chondriosomal starch consists of round granules. Only infrequently there occur locally small grouplets of angular granules of chondriosomal starch.

The intercalations of protein between the starch granules are large. Owing to this, the chondriosomal starch granules are very loosely packed.

The above is a common case of the flinty endosperm of a wheat grain, and fully agrees with Nowacki's (1) explanation of this phenomenon. The



flintiness seems to be due to the presence of large intercalations of protein, which is generally distinguished by transparancy and brittleness when dry.

Fragment *2* also shows a section of the flinty endosperm tissue of emmer wheat. A comparison of this fragment *1* reveals the existence of a considerable difference in their structures. In the second case, the plastid starch granules are, in the majority of cases, relatively very large, but at the same time there occur plastid starch granules, which by their size, form a transition to chondriosomal starch. In the first case, the transition is much less pronounced, the difference in size between the plastid and the chondriosomal starch granules being much more marked.

Unlike the first case, the second is distinguished by the prevalence of the angular chondriosomal starch over the round, the latter forming only an admixture in the mass of the former. Moreover, the chondriosomal starch granules are very closely packed, leaving minutest interspaces for the protein intercalation. Therefore, flintiness is due in this case to the nature of the starch and not to the protein.

The floury tissue of the endosperm of both hard (*Tr. durum* Desf.) and soft (*Tr. vulgare* Host) wheats is built up of a mass of extremely fine and closely packed angular starch granules. The same is true with regard to the endosperm of *Tr. turgidum* L. and *Tr. spelta* L. The properties of the angular chondriosomal starch granules of emmer wheat responsible for the flintiness

of its endosperm seem to be somewhat different from those of the four above-named wheat species. In the emmer wheat, these starch grains are distinguished by their large size.

Fragment *3* shows a section of the floury tissue of the emmer wheat endosperm. The pattern of its mosaic resembles in certain respects that of the mosaic of fragment *1*. The difference in the size of the plastid and chondriosomal starch granules is as marked and the protein intercalations are as large. The granules of the plastid starch are, however, noticeably larger, whilst the chondriosomal starch consists chiefly of angular granules, the round granules, unlike what is observed in fragment *1*, being present in much lower numbers than the angular ones.

The mosaic of fragment *3* shows that the absence of considerable protein intercalations from the endosperm tissue is not an indispensable condition for the production of flouriness. On the contrary, the absence of protein intercalations may sometimes be the cause of the flintiness of the endosperm tissue. All these cases prove that flintiness or flouriness depend chiefly on the properties of the starch and not on the protein component of the contents of the endosperm cells. In a number of cases, the starch may, notwithstanding the similarity in the shape of its granules (angular or round), differ greatly in its optical properties, as well its consistency and other qualities.

The role of the protein in the production of flintiness is, undoubtedly, also far from insignificant.

All the above justifies the conclusion that the flintiness or flouriness of the endosperm tissue of wheat grains is due in different cases to different states of the starch and the protein. There are thus several types of both flintiness and flouriness, due to the great variety in the mosaic pattern of the endosperm, to be observed not only in the different species of wheat, but even in the different races of one species.

In the cereals, distinguished by a simpler and more homogeneous endosperm structure, the combination of structural factors producing flintiness or flouriness must be likewise simpler and more homogeneous. In maize, for instance, in the endosperm of the grains, there are horny and floury tissues, which are in certain respects analogous to the flinty and floury tissues of the endosperm of wheat. The structure is identical in all the different forms and varieties. The horny parts of the endosperm in maize are always built up of closely packed angular starch granules with insignificant protein intercalations, while the floury parts are built up of round loosely arranged starch granules with considerable protein intercalations (3).

The great variety of structural components producing a flinty or floury endosperm structure in wheat is confirmed by fragment *4* of our figure, which represents a part of the flinty endosperm of one of the soft wheats of Chinese origin. The mosaic of this fragment is very complicated. The size of the angular chondriosomal starch grains is varied, the round chondriosomal starch grains are still more varied in size, ranging from very fine granules to such as are indistinguishable in size from small granules of plastid starch.

Although the mosaic of the endosperm tissue, figured in fragment *4* noticeably differs from that of fragments *1* and *2*, it is nevertheless also flinty.

Generally speaking, the flintiness and flouriness of wheat granules are complex phenomena. In its simplest form, independent of the morphology and the internal properties of the starch, flintiness is usually due to a loose arrangement of the starch granules, i. e. to the presence of large protein intercalations. Flouriness is a more complicated phenomenon. Its formation depends to a considerable extent, on the properties of the starch substance as a colloidal mass. Very frequently, the state of the endosperm tissue is to be described as intermediate. In such cases, the essentially flinty endo-

sperm tissue is distinguished by a clearly marked dullness or opalescence, easily observable in grain sections.

It should be emphasized in conclusion that, in the different varieties of wheat, at least, there may occur, in addition to the typical cases of flintiness and flouriness, also other specific combinations of the structural components of endosperm tissue producing flintiness or flouriness.

In other words, one and the same type of mosaic pattern may be found in a wheat variety characterized by a flinty endosperm tissue as well as in that characterized by an endosperm tissue of an intermediate or even floury nature. The reason of this phenomenon undoubtedly lies in the chemical properties of the different structural components.

Received
17. XI. 1937.

REFERENCES

[1] Nowacki, Untersuchungen über das Reifen des Getreides nebst Bemerkungen über den zweckmässigsten Zeitpunkt zur Ernte (1870). [2] Aleksandrova. Aleksandrova, Bulletin of Applied Botany, Genetics and Plant Breeding, Series V A, № 2 (1936). [3] Aleksandrov u. Jakovlev, Journal Botanique de l'URSS, **20** (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

DEVELOPMENT MECHANICS

ATTEMPT AT AN EXPERIMENTAL ANALYSIS OF THE AUXILIARY COMPONENTS OF THE NERVOUS SYSTEM

By N. G. CHLOPIN

(Communicated by I. I. Schmalhausen, Member of the Academy, 10. XI. 1937)

Some two and a half years ago, our Laboratory, while working on the problem of a natural system of histological structures, was faced with the necessity of studying a genetically unified, but at the same time highly multifarious tissue group of auxiliary components of the nervous system. These latter are so diverse that classical histology usually assigns them to different tissues. Thus, the retinal layers, the covering cells of the vascular plexuses of the brain, and, not infrequently, the ependyma, are referred to as «epithelium»; the contractile elements of the iris are as a rule included under the general heading of «muscular tissue»; owing to some of its morphological and functional peculiarities, the neuro glia of the central nervous system is supposed to be closely related to the connective tissue [Kölliker (2), Weigert (6)], and this in spite of its derivation from the neural plate. According to some authors, mainly Spanish ones, the glia of the central nervous system contains, apart from elements of neural origin (astroglia, oligoglia), also some special mesenchymal derivates, the so-called microglia. Schwann's cells, originally regarded by Kölliker and others as elements of the connective tissue, are a peculiar peripheral glia. In the generally accepted view, the so-called «endothelium» or «mesothelium» of the pia mater is a mesenchymal derivate, but, according to the researches of Oberling (5), Harvey and Burr (1), and the opinions of Masson (4) and some other authors, its origin is a neural one.

In studying the textural nature of the auxiliary components of the nervous system under ordinary conditions, one should bear in mind their subordinate position in relation to the neurons, any structures formed by them being dependent on the respective positions and the bonds of the latter with one another. A study of the glia *in vivo*, under pathological or experimental conditions, gives rise to numerous difficulties, owing to the changes taking place simultaneously in the neurons, and the energetic proliferation of the mesenchymal derivates.

It was to be expected that the explantation method, so successful in regard to a number of other tissue types, but not as yet applied in a systematic manner in the analysis of the auxiliary components of the postfetal nervous system, would also in this case help to bring out such histological characteristics as would throw a light on the histologic nature of the glia and the elements genetically related to it. At the present time, we have

at our disposal some evidence enabling us to form an idea of the textural peculiarities of some auxiliary components of the nervous system, such as: the neuroglia of the optic path (Chlopin), of the retinal layers with their contractile derivates (Vinnikov), of the covering cells of the vascular plexuses of the brain (Michailov) and of the covering cells of the pia arachnoidea (Kannegisser). A detailed analysis of the transformations of such varieties of the glia as are known at the present time will be the object of subsequent research.

After a number of preliminary experiments, the optic pathways of young rabbits were selected for the study of the glia of the central nervous system, this being a portion of the white matter containing no neuron bodies and rather slow in undergoing myelinization. The growth of particles in various parts of the optic paths was studied *in vitro* for one month and a half. The mesenchymal elements, which began to grow in the course of a few days upon explantation, were subsequently crowded out by the vigourously proliferating neuroglia, and, as a rule, completely disappeared. The growth of the glia involved a strong liquefaction of the coagulated fibrin, and not infrequently the formation of cavities in the nutrient medium and the deformation of the zone of growth. The whole mass of the glia took part in the formation of the zone of growth. According to Marchesini (3), the glia consists first of all of fibrous astroglia and oligodendroglia, the growing elements of which become indistinguishable from one another. The growth of the glia *in vitro* is a complex one, although loose reticular structures or variously shaped isolated cells may be produced at the same time. The earlier stages of cultivation are characterized by the appearance of large numbers of free rounded or ameboid glial elements. At all the stages of cultivation, the glial new. growth preserves its distinctive character, which makes it easy of identification, as it cannot be easily confused with either the mesenchymal or the various epithelial explantates. It is the growing glia that is characterized by the rather massive, and not infrequently epithelium-like layers, by peculiar cellular strands and flat membranes. The presence of all these formations often gives the explantates a star-shaped or radial appearance. Glial membranes mostly consist of elongated elements forming palisade-like or vortical structures. Such portions of glial membranes as consist of polygonal epithelium-like elements are rather rare.

The behaviour of the glia throughout the cultivation period is that of a completely determined peculiar tissue. Large numbers of argyrophile fibres may be observed to form in the zone of growth of pure glial cultures. They are in all probability peculiar structures qualitatively differing from the mesenchymal argyrophile fibres.

Such pictures of growth and such histoblastic potencies of the auxiliary components of the nervous system under consideration as have been described by either my coworkers or myself, necessarily possess many features in common, since all the objects investigated produce structures characteristic of the glia. On the other hand, some of their characteristics make it possible to arrange them in a series, with the covering cells of the vascular plexuses of the brain and the retinal layers with their contractile derivates at one end, and the glia of optic paths and the covering cells of the pia mater at the other. The former, if kept in tissue cultures, give rise not only to complex, but also, not infrequently, to flat epithelium-like structures. The latter show only a weakened tendency to form such epithelium-like structures, while they retain a high degree of complexity. One is struck by the fact that the covering cells of the pia mater give rise to growths practically undistinguishalbe from neuroglial cultures proper and quite dissimilar from mesenchymal explantates. This is, in our opinion, a convincing proof of their

being genetically related to the elements of the neuroglia, i. e. of their having originated from the neural germ.

Such evidence as we now possess regarding the growth and the transformations of Schwann's cells of peripheral nerves in tissue cultures show that these cells, while undoubtedly differing from mesenchymal derivates, have to a great extent lost their ability to form complex structures.

All these facts, if placed side by side with the evidence afforded by comparative anatomy and embryology, would warrant the assumption that the auxiliary components of the nervous system have been philogenetically developed in chordates from the dorsal portion of the ectoderm of the ancestors, in which all the elements of the nervous system were concentrated and which possibly acted as a sense organ, and also from placode-like formations with receptor neurons.

Upon the formation of the neural tube, the development of such portion of the former epidermic covering assumed a different direction, tending to become a peculiar tissue of the inner medium of the body, whose special purpose was to serve the nervous system. This was bound to result in some loss of epidermic characteristics and the acquisition of specific glial characteristics. The auxiliary components of the central nervous system retained, however, although in a modified state, their importance as a barrier in relation to the general internal mesenchymal medium of the organism. The greatest secondary changes took place in the peripheral glia, and, more particularly, in Schwann's cells, which lost to a great extent their ability to form complex structures and apparently also their importance as a barrier. The ability of glial elements to form argyrophile structures may be supposed to be a manifestation of a property common to all living protoplasm, namely the ability to form fibrous structures of mechanical importance.

General Morphology Section,
A. M. Gorky All-Union Institute of Experimental Medicine.
Cytological Section, Oncological Institute.
Leningrad.

Received
10.XI.1937.

REFERENCES

[1] Harvey a. Burr, Arch. of neurol. and psych., **15** (1926). [2] Kölliker, Handbuch der Gewebelehre des Menschen, I (1889). [3] Marchesini, Graefes Arch., **117** (1926). [4] Masson, Diagnostics de laboratoire, II. Tumeurs (1923). [5] Oberling, Bull. Assoc. franç. pour l'étude du cancer, **2** (1922). [6] Weigert, Beiträge zur Kenntniss der normalen menschlichen Neuroglia (1895).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

DEVELOPMENT MECHANICS

GROWTH AND TRANSFORMATION *IN VITRO* OF MYONEURAL ELEMENTS (SPHINCTER AND DILATATOR OF THE IRIS)

By J. A. VINNIKOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 10. XI. 1937)

Studies of the contractile elements of the iris were a logical sequel of a recently published work by the same author dealing with the retinal layers of the edge of the eyecup, whose derivates they are. Owing to the external similarity of these structures to the smooth mesenchymal muscles they had until quite recently been included into one group (Häggquist).

The object of the present paper is to study the histogenesis of the sphincter and the dilatator, to compare the details of their structure with mesenchymal muscles and, mainly, to bring out the peculiarities of theses structures, as derivates of the neural plate, in tissue cultures.

Eyes of rabbits, both pigmented and albinotic, were used in these experiments, as well as pig embryos. The explantates were studied *in vitro* for two months.

The sphincter of the iris is developed from a fold of retinal layers, the so-called club-shaped body, in the elements of which myofibrils appear. No mesenchyma or blood vessel grow among the differentiating cells of the sphincter. In the cells of non-albinoes, separate pigment grains are, as a rule, preserved. The preparations treated after Foot disclose in the sphincter argyrophil fibres, which differ, by their appearance and arrangement, from the precollagenic structures of smooth mesenchymal muscles, as well as from muscles of another origin. Neither the penetration into the sphincter, nor the presence therein of precollagenic structures, or of collagenic and elastic fibres from the neighbouring stroma, could be traced. In tissue cultures, the elements of the sphincter become more and more separated, the differentiated fibrils vanish, and large amounts of pigment appear in the cytoplasm of non-albinoes. These elements of the sphincter *in vitro* are reminiscent of the early stages of its histogenesis *in vivo*. The modified cells of the sphincter either penetrate into the zone of growth, or remain for a long time within the explantate, where, at later stages, they become practically indistinguishable from the elements of the retinal layers of the iris. The argyrophil fibres within the non-growing elements of the explantate are destroyed.

Explantation causes the elements of the dilatator of the iris to be modified in a similar way.

The elements of the sphincter and the dilatator, having penetrated into the zone of growth, form varied structures in the shape of branching strands,

membranes, flat reticular formations and isolated cells. Neither the transformation of these structures nor the fate of the explantate differ in any way from the evidence obtained by the writer in the explantation of retinal layers of the iris and of ciliary processes.

The dilatator and the sphincter of the iris, being genetically related to the neural plate, differ by their properties from the muscles of mesenchymal or mesodermic origin, not excepting the myocardium. The contractile elements of the iris are myoneural elements of mammals, occupying as they do a peculiar position in the collective group of contractile tissues, while their origin, their position in the eyecup, as well as their histoblastic potencies, place them among the auxiliary components of the central nervous system.

The evidence yielded by the present work confirms the point of view of Prof. N. G. Chlopin, according to which the natural system of histological structures should be based on the genetic principle.

General Morphology Section,
A. M. Gorky All-Union Institute of Experimental Medicine. Moscow.
Cytological Department, Oncological Institute.
Leningrad.

Received
10.XI.1937.

REFERENCES

[1] Винников, Арх. анат., гист. и эмбр., **15** (1936). [2] Häggquist, Handb. d. mikr. Anat., **2**, III (1939). [3] Хлопин, Арх. анат., гист. и эмбр., **15** (1936); ZS. mikr.-anat. Forschg., **34** (1933).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

DEVELOPMENT MECHANICS

GROWTH AND TRANSFORMATION *IN VITRO* OF THE COVERING CELLS OF THE VASCULAR PLEXUSES OF THE BRAIN

By V. P. MICHAILOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 10. XI. 1937)

The vascular plexuses of the brain, which are pia-arachnoidal outgrowths containing numerous blood vessels, are covered with one layer of peculiarly modified ependymal cells, the fine structure of which places beyond doubt their glandular character. The explantation of covering cells of the plexuses was of interest not only because no work of the kind had been done before, but also because these elements, the morphological and functional characteristics of which make of them as it were a typical glandular epithelium (a name they are usually given in all original research works, synopses and manuals), are undoubtedly of neural origin. A study of the general aspects of the growth and the transformations *in vitro* of the covering cells of the vascular plexuses of the brain, as well as a comparison of the results obtained with the literary evidence available, was therefore of paramount interest.

The explantation of the covering cells of vascular plexuses of the brain of rabbits (from embryo up to adults) involved considerable difficulties, since, owing to their extraordinary proteolytic power, the coagulated fibrin was constantly liquefied. Nevertheless, in about 20% of the experiments (30 series of experiments involving about 500 cultures were made) a satisfactory growth was obtained.

The covering cells divide mitotically in an energetic manner and form a zone of growth. The range of forms resulting from their growth is very wide, so that, side by side with epithelium-like membranes, there are formed complex structures that usually characterize the auxiliary elements of the nervous system. Frequently, the covering cells of the plexuses grow as narrow branching strands diverging both from the explantate itself and secondarily from the membranes, sometimes in another plane. Owing to a continual formation of cavities due to liquefaction some groups of cells are pushed away from the primary piece and can produce secondary star-like growths or membranes with serrated edges. A marked characteristic of the cellular complexes of the zone of growth is the fibrillation of their protoplasm which is especially noticeable in a series of sections when stained with iron hematoxylin. In the zone of growth there are sometimes formed thick three-dimensional outgrowths consisting of concentrically arranged cells somewhat reminiscent of pearls and characteristic of tissues of the epidermal type. No epithelization of the explantate takes place. This type of growth, followed

up on a number of cultures up to an age of 34 days, cannot in any case be compared with the type of growth of ordinary epithelia, as it differs profoundly therefrom.

The striking similarity displayed by the cultures of the covering cells of the vascular plexuses of the brain with those of the iris, the ciliary processes and the myoneural elements of the eye [Vinnikov (1)], and those of the glial elements of the optic paths [Chlopin (2)], points to the relationship between all these elements so different as to structure and functions, but having the same neural origin. On the other hand, the cultures of the covering cells of the brain plexuses differ from glial cultures in that the structures formed by them display a greater similarity to the epithelium.

General Morphology Section,
A. M. Gorky All-Union Institute of Experimental Medicine. Moscow.
Cytological Section, Oncological Institute.
Leningrad.

Received
10. XI. 1937.

REFERENCES

[1] Винников, Арх. анат., гист. и эмбр., **15** (1936). [2] Хлопин, C. R. Sci. URSS., present fascicle.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

DEVELOPMENT MECHANICS

GROWTH AND TRANSFORMATION *IN VITRO* OF THE ELEMENTS OF THE PIA MATER

By N. N. KANNEGISSER

(Communicated by I. I. Schmalhausen, Member of the Academy, 10. XI. 1937)

The origin of the covering elements of the pia mater is a controversial question. Most of the modern investigators, in common with the classics of histology, uphold the theory of their mesenchymal origin. At the same time, a number of American and French authors, upon analysing pathological data as well as the evidence yielded by experimental embryology, regard these elements as derivates of the neural germ.

Having regard to the general importance of the method of tissue cultures as applied to the study of problems of histogenesis, an investigation of the growth and transformation of these elements *in vitro* was of some interest.

The cultures of the tissues of the pia mater of the rabbit, followed up for 35 days upon explantation, have displayed a type of growth absolutely dissimilar from that of mesenchymal tissues repeatedly described as a grass-like growth. A characteristic feature of the growth of the pia mater is the complexity displayed in the formation of membranes and peculiar strands. An energetic liquefaction of the medium causes the membranes to break into separate parts, which may become new centres of growth.

The growth of connective tissue elements that is noticeable in the early stages of cultivation is usually ultimately crowded out by the energetically proliferating covering cells. Some microscopic sections make it possible to trace the direct penetration of covering cells into the zone of growth. Growth in several planes, a palisade-like arrangement of the nuclei and the formation of vortical structures are the main characteristics of the growth *in vitro* of these elements.

The type of growth displayed by the covering cells of the pia mater of the brain reproduces with such fidelity the growth of the glia of optical paths that one may affirm with assurance that the growth of the elements under investigation is of the glial type. From our point of view, this fact is a direct proof of the above-mentioned conception of American and French

authors [Harvey and Burr (1), Oberling (3), Masson (2)] as to the neural origin of the covering cells elements of the pia mater.

General Morphology Section.
A. M. Gorky All-Union Institute of Experimental Medicine. Moskow.
Cytological Section, Oncological Institute.
Leningrad.

Received
10. XI. 1937.

REFERENCES

[1] Harvey a. Burr, Arch. of neurol. and psych., 15 (1926). [2] Masson, Diagnostics de laboratoire, II. Tumeurs (1923). [3] Oberling, Bull. Assoc. franç. pour l'étude du cancer, 2 (1922).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1938. Volume XVIII, № 2

ENTWICKLUNGSMECHANIK

EKTODERM DER ANURENGASTRULA UNTER INDUKTIONSWIRKUNG ABGETÖTETER ORGANISATOREN UND ORGANTEILE VON AMPHIBIEN UND MAMMALIEN

Von G. A. SCHMIDT

(Vorgelegt von I. I. Schmalhausen, Mitglied d. Akademie, d. 26. X. 1937)

Bekanntlich ist bei *Triton taenitaus* die chemische Natur der Organisatorwirkung nachgewiesen: zum ersten Male als Andeutung in den Arbeiten von Spemann (1931) (1), welcher, nach Zerquetschung oder Austrocknung, durch Einsteckung des Organisators Hinweise auf Induktionserscheinungen im Gastrulaektoderm erhielt. Dann von vier anderen Autoren, H. Bautzmann, Holtfreter, E. Wehmeier, O. Mangold (1932) (2), welche die Tatsache der Induktion mittels toter Organisatoren nachwiesen. Holtfreter (1933) (3) machte eine Reihe von Versuchen bei denen er, erstens, zeigte, dass der durch Kochen oder Austrocknen abgetötete Organisator seine Induktionsfähigkeiten nicht ganz verliert, zweitens, dass auch andere Teile des Embryos, die in lebendem Zustande keine Induktionen hervorrufen, nach dem Abtöten induzierend wirken, und, drittens, dass verschiedene Gewebe und Organe der Tiere zur Induktionswirkung fähig sind, was auf die weite Verbreitung der induzierenden Stoffe im Tierreiche hinweist (1934) (3). M. N. Ragozina (1936) (4) zeigte dass auch pflänzliche Gewebe Induktionswirkung ausüben.

Im Jahre 1934 unternahm ich einige Versuche an Anurenkeimen, die als Parallele zu Versuchen an Tritonkeimen betrachtet werden können. Diese Versuche wurden auch im Jahre 1935 weitergeführt und ergaben einige interessante Tatsachen, über welche ich hier kurz berichten möchte.

Als Untersuchungsmaterial dienten am besten Embryonen der bunten Kröte (*Bufo viridis*), aber auch solche von *Rana esculenta*. Embryonen von *Bufo viridis* sind leicht operierbar, sie haben nicht so dickes Ektoderm wie die anderen Anurenarten.

Es wurden Versuche mit abgetöteten Organisatoren angestellt, die zeigten, dass Stücke des Urmundes, bei 60 Grad getrocknet, sowie Stücke, die in 95%-igem Alkohol fixiert worden waren, deutliche Induktionen bewirken.

In einer Versuchsserie wurden schöne Induktionen der Kopfsinnesorgane mittels Implantation von Stücken der Medullarplatte erzielt, was die Induktionsfähigkeit dieses Embryoteiles zeigt. Aber diese Versuche ergaben die Feststellung einer assimilatorischen Induktion bei Anuren, so dass also diese keine spezielle Beziehung zur Frage der chemischen Natur der Induktionswirkung aufweisen.

Es wurden weiter verschiedene Serien der Implantation von Teilen der Organe der Kaulquappe und der erwachsenen Mammalien gemacht. Bei

der Implantation von Teilen der Amphibienorgane wurden die besten Induktionen bei Implantation von Muskeln, Leber und Gehirn hervorgerufen. Bei Mammalien erwiesen sich Niere und Leber als stark induzierend.

Es war interessant zu prüfen, ob die Organteile der Mammalien grössere Induktionswirkung ausüben würden als die Organe der Amphibien, da Holtfreter darauf hingewiesen hatte, dass die Gewebe der Mammalien viel grössere Induktionsfähigkeiten besitzen als die der Amphibien. Da bei Anuren Induktionen überhaupt viel schwerer hervorzurufen sind, so gab es hier eine gute Möglichkeit zu prüfen, ob ein klarer Unterschied in den Induktionsfähigkeiten zwischen den Geweben der Mammalien und Amphibien festzustellen ist.

Die entsprechenden Experimente gaben eine positive Antwort, welche auch bei diesem Materal die Angaben Holtfreters über Triton bestätigte. Die Gewebe der Mammalien sind viel induktionsbefähigter als die der Amphibien.

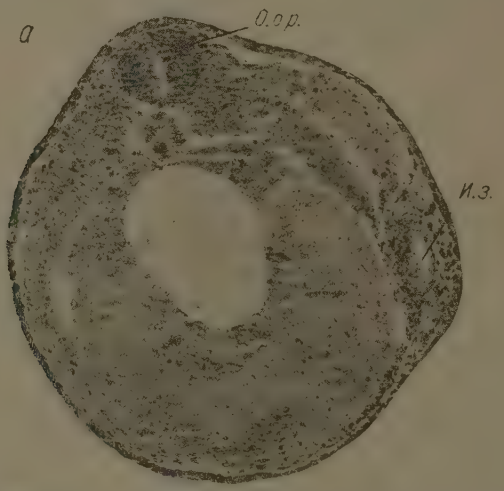


Abb. 1a. Ein Schnitt des Keims 13 *Bufo viridis*, 1935, welchem ins Blastocoel der Gastrula ein Stück einer Rattenniere eingesteckt wurde. Schwache Vergrösserung. *O. o. p.*— Achsenorgane des Wirts, *H. s.*—induzierte Anlage; unter der Medullarplatte liegt keine Chorda, Vergrösserung 60.

Die grössere Wirkung konnte nach dem grösseren Prozentsatz der induzierten Fälle festgestellt werden. Was den Bau der induzierten Anlage betrifft, so muss man an dieser Stelle das wiederholen, was schon in anderem Zusammenhange gesagt wurde (1936) (5): Die Induktion hat bei Anuren wegen der verschiedenen Eigenschaften des induzierbaren Materials viel grössere Schwierigkeiten als bei Urodelen. Dadurch erklärt es sich, dass ich nie so grosse induzierte Anlagen bekam, wie Holtfreter sie bei Triton nachgewiesen hat. Dazu muss ich noch folgendes bemerken. Mitunter erhielt ich auch eine grosse, scheinbar induzierte Anlage (z. B. Fall 104, *Bufo viridis*, 1935), bei der aber keine unmittelbare Induktionswirkung des Implantats vorhanden war, sondern nur eine mechanische Wirkung auf das chorda-mesodermale Material, welches zur Spaltung des Urdarmdaches führte und auf diese Weise die sekundäre Anlage ergab.

Der Unterschied zwischen solchen Fällen scheinbarer Induktionen und denen echter Induktionen mittels Organteilen oder toter Organisatoren be-

steht darin, dass, während in letztgenannten Fällen nie eine Chorda erscheint, in den ersteren auch Chorda vorhanden sein kann.

Ein Fall echter Induktion mittels Organteilen ist auf Abb. 1a, b, dargestellt. Hier wurde ein Stückchen Rattenniere in das Blastocoel einer *Bufo viridis* Gastrula implantiert, und dadurch erfolgte die Induktion einer Medullarplatte. Die Medullarwülste waren gut entwickelt, und nach Verschluss derselben wurde der Embryo fixiert und untersucht. Auf den Schnitten kann man feststellen, dass ein Neuralrohr mit einer daneben-liegenden, von diesem induzierten Hörblase induziert worden ist. Die Induktion ist typisch für eine nicht von lebenden Induktoren hervorgerufene. Der Bau des Medullarrohres ist etwas atypisch, es fehlt die für ein normal induziertes Röhrchen typische Gliederung des ganzen Gebildes.

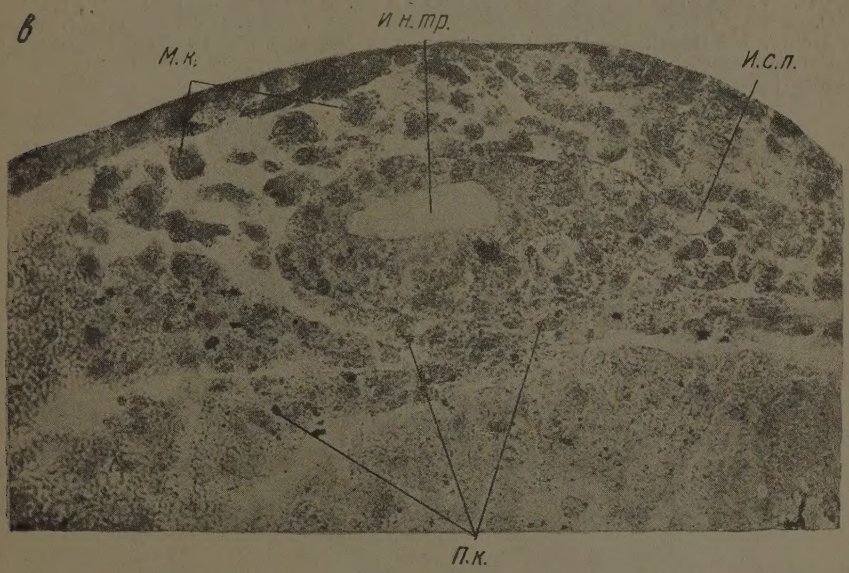


Abb. 1b. Die induzierte Anlage des Keims 13 *Bufo viridis* bei grösserer Vergrösserung. *М. К.*—Mesenchymzellen, reich entwickelt, *И. н. тр.*—induziertes Medullarrohr, atypisch gebaut, *И. с. п.*—induzierte Hörblase, *П. к.*—Reste der eingesteckten Rattenniere. Vergrösserung 150.

Diese Angaben bestätigen die von Holtfreter an *Triton taeniatus* gemachten Versuche.

Was aber die theoretische Bedeutung der Fälle anbelangt so erinnere ich an einen Ausspruch der Schweizer Forscher, die sich mit der Frage nach der Bedeutung des Nervernsystems für die Regeneration befasst haben. Nach ihrer Auffassung ist das Nervensystem «Facteur nécessaire, mais pas suffisant», ein notwendiger, doch nicht genügender Faktor. Die chemische Induktion ist ein notwendiges Glied in der Kette der Erscheinungen, die zur Bildung des Medullarrohres führen, doch ist diese Induktion nicht genügend, um allein eine vollkommene, normal entwickelte und gegliederte Embryonalanlage hervorzurufen.

Die Tatsache, dass die Einwirkung bei der Induktion der Organe chemischen Charakter hat, ist deshalb so wichtig, weil damit gezeigt ist, dass der Einfluss eines Teiles auf einen anderen stofflichen Charakter hat. Die Notwendigkeit der Untersuchung der rein biologischen Gesetzmässigkeiten,

solcher wie Regionalität, Proportionalität u. s. w. bleibt bestehen. Wir brauchen nicht das Biologische auf das Chemische zurückzuführen, auch wenn wir die chemische Natur der Induktionswirkung feststellen. Es scheint mir wichtig, diese Bemerkung zu machen, weil in der Frage über die chemische Natur der Induktion sonst leicht vulgär-mechanistische Vorstellungen entstehen.

Institut für experimentelle Biologie.
Moskau.

Eingegangen
d. 28. VIII. 1937.

ZITIERTE LITERATUR

[1] H. Spemann, Verh. Dtsch. Zool. Gesellsch. Utrecht, **34**, 129—132 (1931). [2] H. Bautzmann, J. Holtfreter, H. Spemann (Wehmeier) u. O. Mangold, Naturwiss., 20 Jahrg., H. 51 (1932). [3] J. Holtfreter, Roux Archiv, **128** (1934); **132** (1934); **132**, H. 2 u. 3, 225—306. [4] М. Н. Раговина, Биолог. журн., V, № 6 (1936). [5] G. A. Schmidt, C. R. Acad. Sci. URSS, IV, № 9 (1936); Zool. Anz., **116**, H. 11/12, 323—330 (1936).

PRIX D'ABONNEMENT

Un an—36 livraisons . . . . . $ 15
Six mois — 18 livraisons . . . $ 7.50
(Séparément $ 0.50 le numéro)

URSS — Meždunarodnaja Kniga. 18 Kuznetskij Most, Moscou (Compte courant n° 263 Banque d'Etat de l'URSS, 12, Neglinnaja, Moscou),
U. S. A., NORTH and SOUTH AMERICA — BOOKNIGA CORPORATION. 255 Fifth Avenue. New York, N. Y.
ENGLAND — W. H. SMITH & son, Ltd Strand House, Portugal Sat London, W. C. 2.
FRANCE — MESSAGERIES HACHETTE. Service Abonnements. 111, Rue Réaumur. Paris, 2e.
SUISSE — Administration du Journal „LE TRAVAIL". 39, Rue de Lausanne. Genève.
Č. S. R. — MELANTRICH, Akc. Spol. Knihkupectvi — odd. slovanskych knih. Vaclavské nam 42. Praha II. (Ucet post. spor. 20. 208).
BELGIQUE — LIBRAIRIE A. U. S. 38 Vieille Halle-aux-Blés. Bruxelles (C. C. P. 1861. 15). (P. Driessen)
CHINA — AMERICAN BOOK & SUPPLY Co 841 Bubbling Well Road. P. O. Box 1798. Shanghai.
HOLLAND — J. GINSBERG. University Bookseller. Kort Rapenburg, 17. Leiden
LIETUVA — „MOKSLO". Knygynas. Laisves Aleja, 46. Kaunas.
TURQUIE — LIBRAIRIE HACHETTE. Succursale de Turquie. 469, Avenue de l'Indépendance. Boite Postale Pera 2219. Istanbul.
DEUTSCHLAND — OTTO HARROSSOWITZ. Leipzig C. I. Auerstrasse 14.
POLSKA — GEBETHNER & WOLFF. Filja l. Krakowskie Przedm. 15. Warszawa.
ROSSICA. Chmielna 5. Warszawa.
PALESTINE — SAFRAN Co Jaffa Rd. 36. P. O. B. 223. Haifa.
LATVIJA — VALTERS un RAPA Akc. Sab. Teatra iela II. Riga (Pasta tek. rek. № 675).
BULGARIA — T. F. TSCHIPEFF Buchhandlung Boulevard Dondoukoff 20/22 Sofia. (Postscheckkonto-Sofia 540).
NORGE — JOHAN GRUNDT TANUM. Tidligere Aschehougs Boghandel. Karl. Johans gt. 43. Oslo.

Les «Comptes Rendus», largement expédiés à l'étranger dans les centres scientifiques les plus importants et dans les bureaux de nombreuses revues, sont le seul organe scientifique en URSS, donnant tous les dix jours un compte rendu des plus récentes acquisitions de la science soviétique.

Prix — $ 0.50